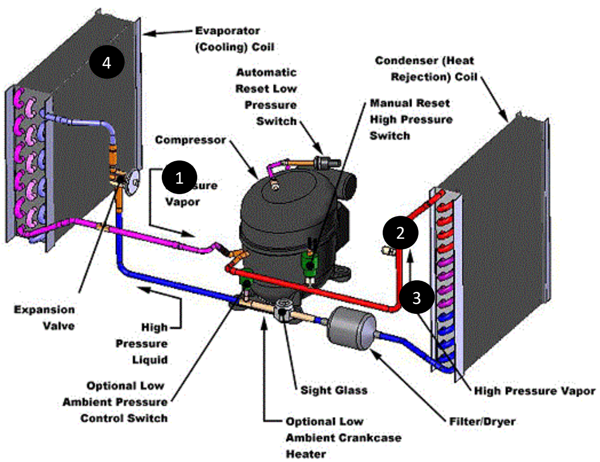
Fault estimation and Fault-tolerant control of vapor compression cycle systems

JAEPIL BAN, WOOKYUNG KWON, SANGCHUL WON

POSTECH, KOREA

Contents

What is VCC systems?



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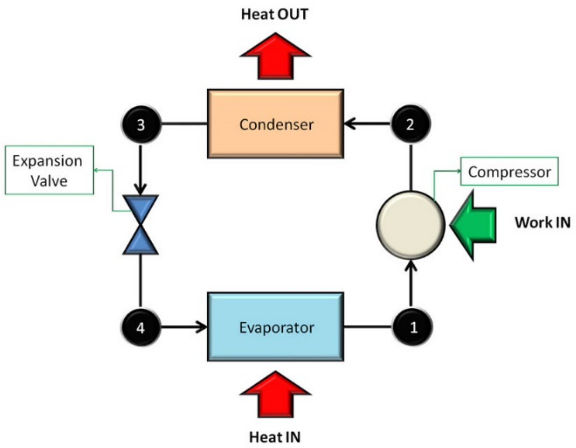

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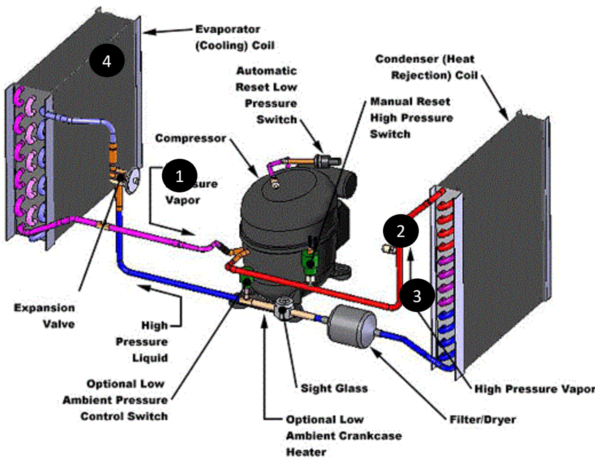

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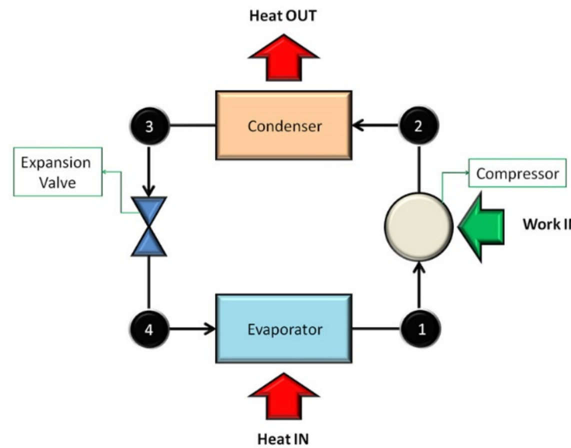

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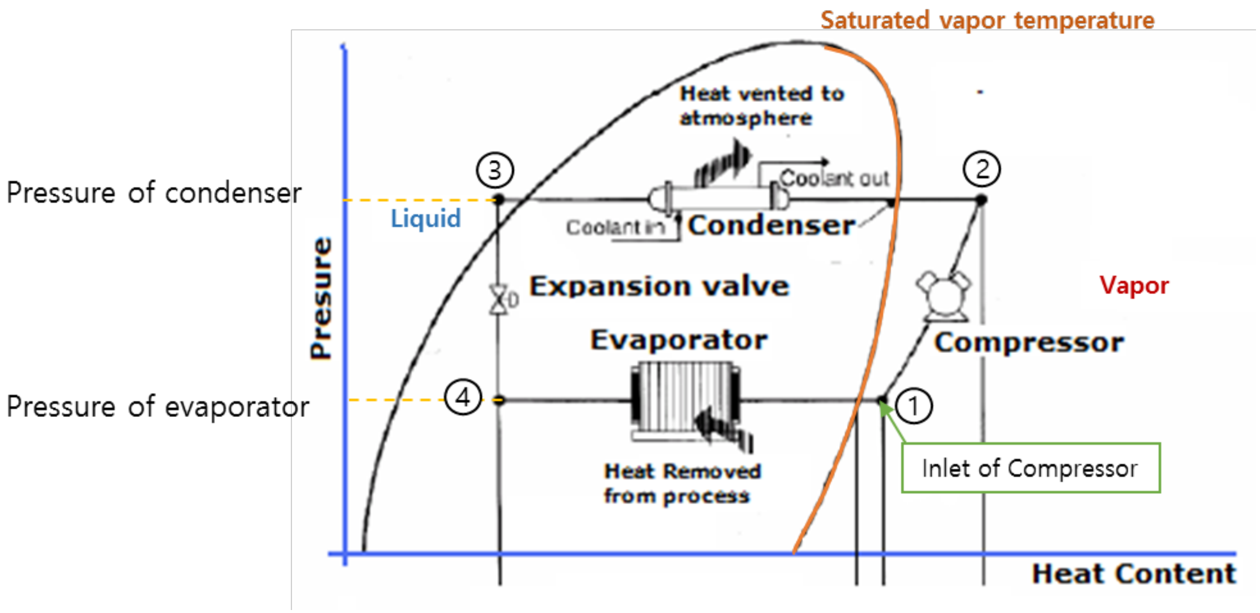
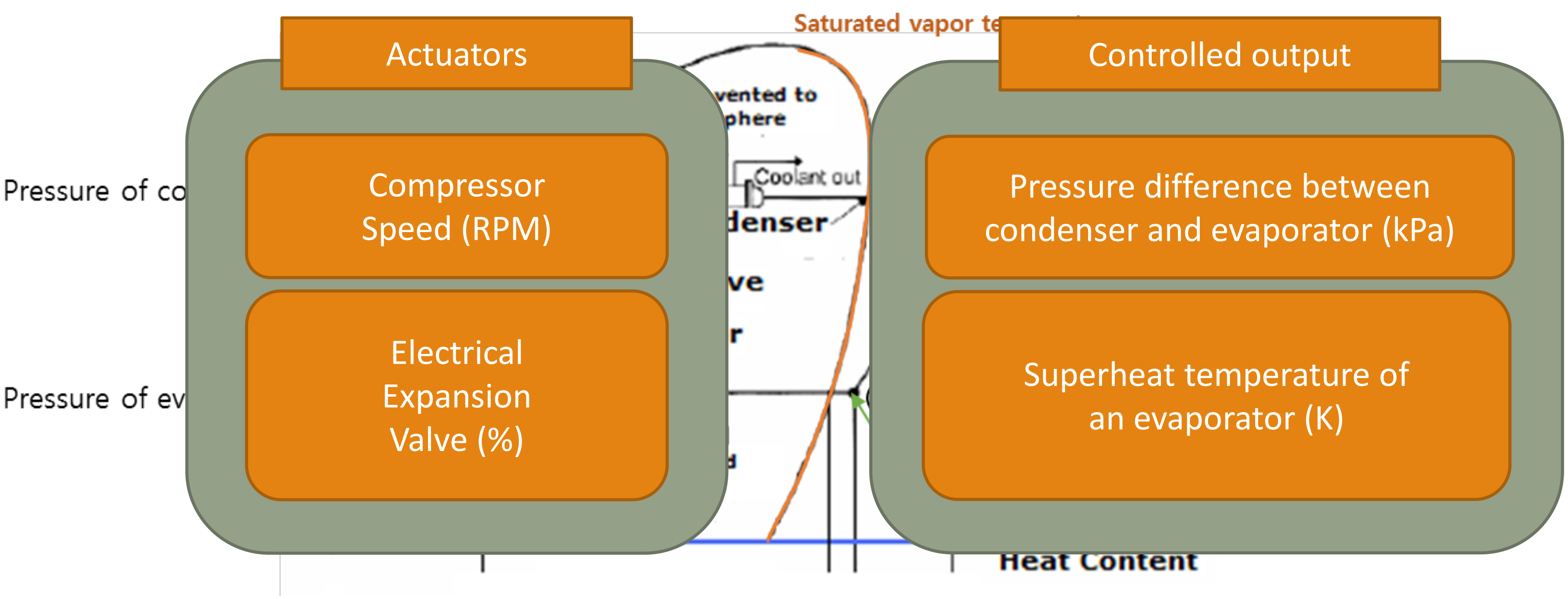
applications

Pressure and superheat temperature are important thing to be controlled



Superheat temperature: temperature of a vapor above the saturated vapor temperature.

Pressure and superheat temperature are important thing to be controlled



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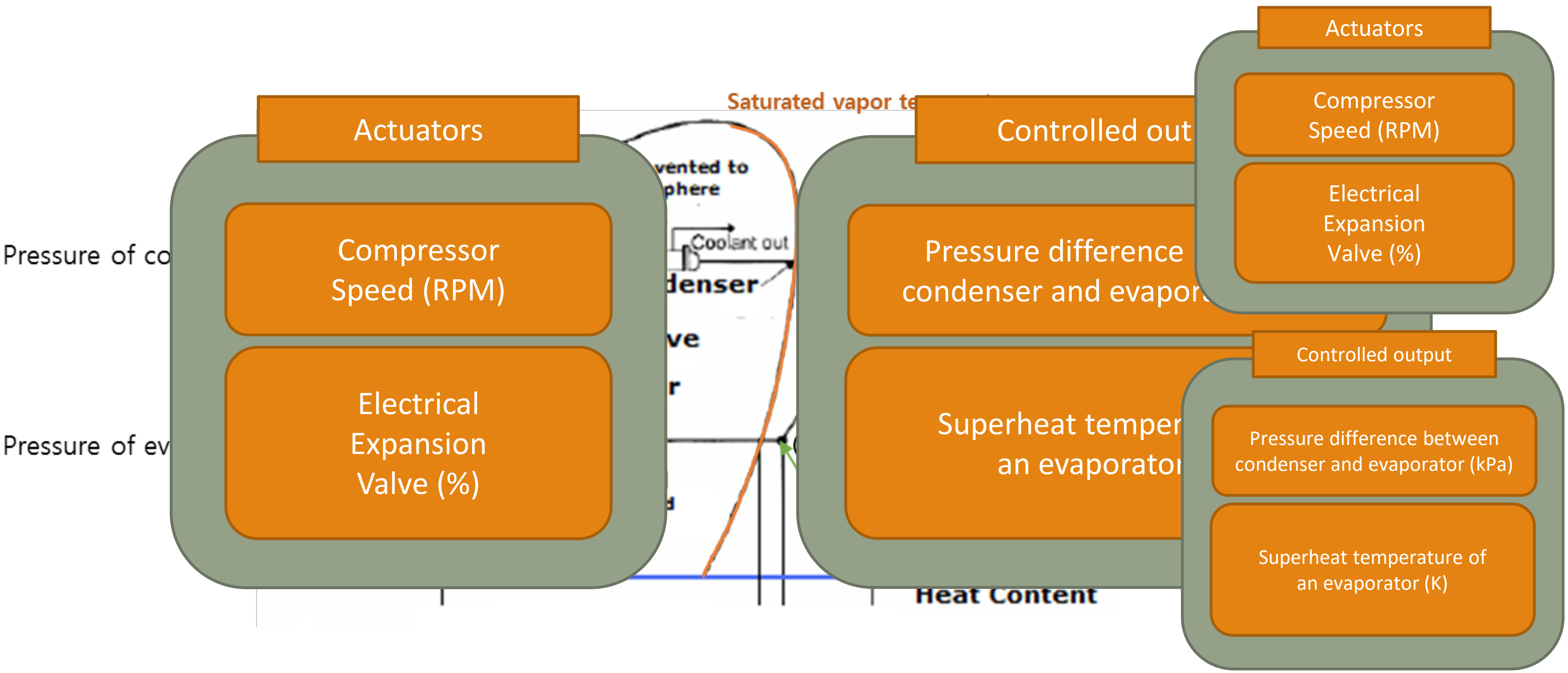

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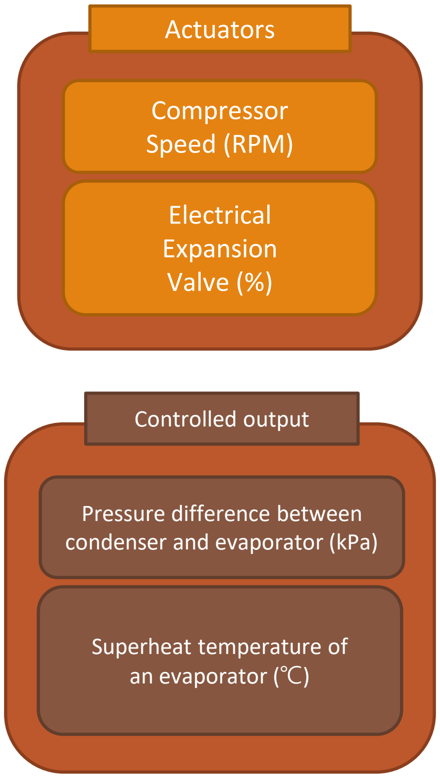
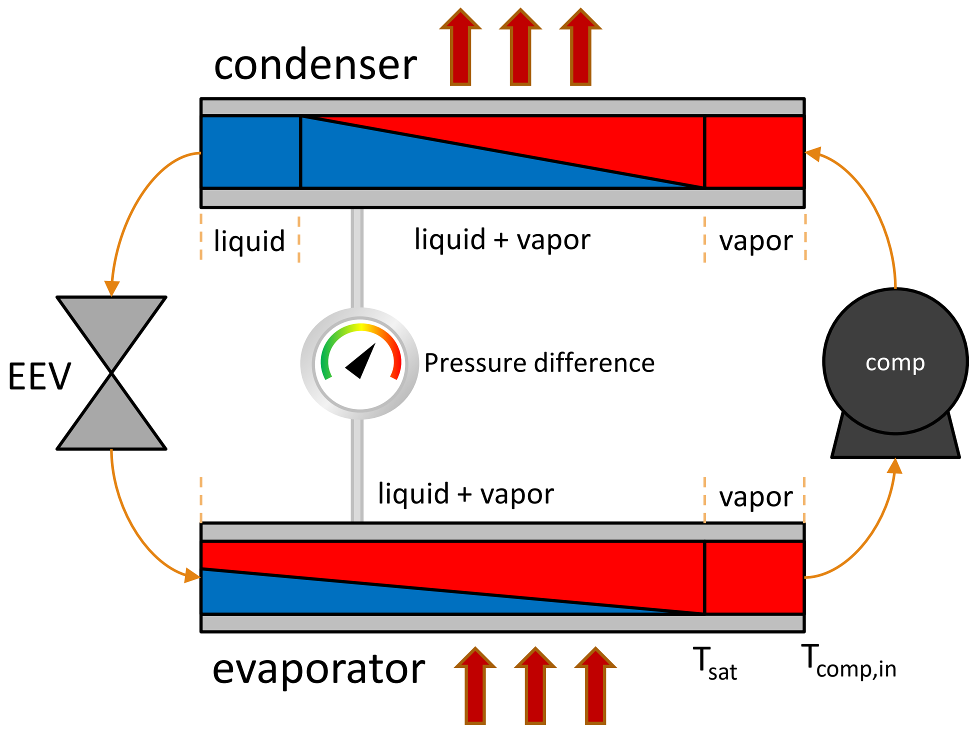




Superheat temperature: temperature of a vapor above the saturated vapor temperature($T_{comp,in} - T_{sat}$).

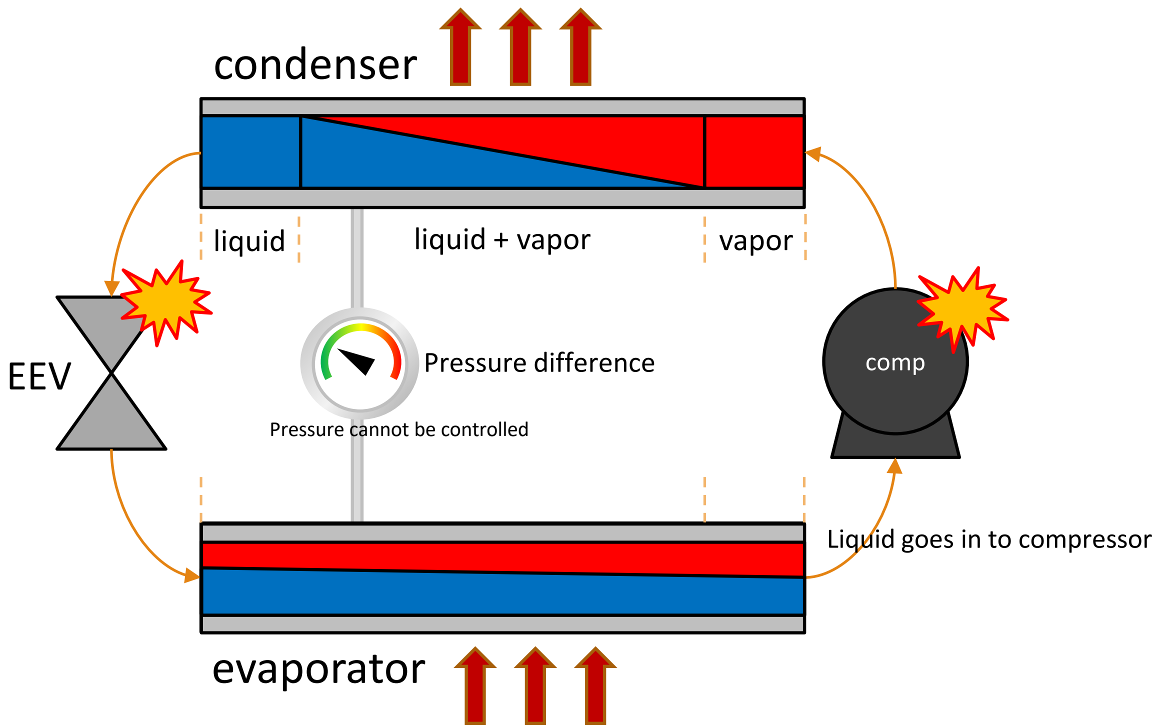
Fault cause severe damage to the system

Actuators

- Compressor Speed (RPM)
- Electrical Expansion Valve (%)

Controlled output

- Pressure difference between condenser and evaporator (kPa)
- Superheat temperature of an evaporator (℃)



Superheat temperature: temperature of a vapor above the saturated vapor temperature($T_{comp,in} - T_{sat}$).

Nonlinear model for VCC system

Static Models

- Compressor model

$$\begin{aligned}\dot{m}_k &= \rho_k V_k w_k \eta_{vol}, \\ h_{out} &= \frac{1}{\eta_k}\left[h_{out,isentropic} + h_{in}\right](\eta_k - 1)\right],\end{aligned}$$

- Electrical expansion valve

$$\begin{aligned}\dot{m}_v &= C_d\sqrt{\rho_v\left(P_{in} - P_{out}\right)}, \\ h_{v,in} &= h_{v,out},\end{aligned}$$

Dynamic models

- Condenser

$$Z_c(x_c)\dot{x}_c = f_c(x_c, u)$$

$$\begin{aligned}x_c &= [L_{c1} \quad L_{c2} \quad P_c \quad h_{cout} \quad T_{wc1} \quad T_{wc2} \quad T_{wc3}]^T \\ u &= [w_k \quad u_v \quad \dot{m}_{ea} \quad \dot{m}_{ca} \quad T_{ea,in} \quad T_{ca,in}]^T\end{aligned}$$

- Evaporator

$$Z_e(x_e)\dot{x}_e = f_e(x_e, u)$$

$$\begin{aligned}x_e &= [L_{e1} \quad P_e \quad h_{eout} \quad T_{we1} \quad T_{we2}]^T \\ u &= [w_k \quad u_v \quad \dot{m}_{ea} \quad \dot{m}_{ca} \quad T_{ea,in} \quad T_{ca,in}]^T\end{aligned}$$

Nonlinear model for VCC system

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- Evaporator

$$Z_e(x_e)\dot{x}_e = f_e(x_e, u)$$

$$x_e = [L_{e1} \quad P_e \quad h_{eout} \quad T_{we1} \quad T_{we2}]^T$$
$$u = [w_k \quad u_v \quad \dot{m}_{ea} \quad \dot{m}_{ca} \quad T_{ea,in} \quad T_{ca,in}]^T$$

- Nonlinear descriptor form

$$\begin{bmatrix} Z_c(x_c) & 0_{7\times5} \\ 0_{5\times7} & Z_e(x_e) \end{bmatrix} \begin{bmatrix} \dot{x}_c \\ \dot{x}_e \end{bmatrix} = \begin{bmatrix} f_c(x_c, u) \\ f_e(x_e, u) \end{bmatrix},$$

$$y = g(x_c, x_e) = [\Delta P \quad SH]^T, \quad u = [w_k \quad u_v \quad \dot{m}_{ea} \quad \dot{m}_{ca} \quad T_{ea,in} \quad T_{ca,in}]^T.$$

Linearization of Nonlinear model

Given nonlinear model

$$
\begin{aligned}
Z(x,u)\dot{x} &= f(x,u) \\
y &= g(x)
\end{aligned}
$$

$$
\begin{aligned}
Z(x,u)\dot{x} &= f(x,u) \\
\dot{x} &= Z(x,u)^{-1} f(x,u) \\
&= F(x,u).
\end{aligned}
$$

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Setting an operation point

$$x = x_o + \delta x$$
$$u = u_o + \delta u$$

$$\delta\dot{x} = \left[\left.\frac{\partial F}{\partial x}\right|_{x_o,u_o} \right] \delta x + \left[\left.\frac{\partial F}{\partial u}\right|_{x_o,u_o} \right] \delta u$$

expanding

$$\left.\frac{\partial F}{\partial x}\right|_{x_o,u_o} = Z(x_o)^{-1} \left.\frac{\partial f}{\partial x}\right|_{x_o,u_o} + \left.\frac{\partial Z^{-1}}{\partial x}\right|_{x_o} f(x_o,u_o)$$

$$f(x_o,u_o) = 0$$

Linearization of Nonlinear model

Given nonlinear model

$$\begin{aligned} Z(x,u)\dot{x} &= f(x,u) \\ y &= g(x) \end{aligned}$$

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Setting an operation point

$$x = x_o + \delta x$$
$$u = u_o + \delta u$$

$$\dot{\delta x} = \left[\left. \frac{\partial F}{\partial x} \right|_{x_o,u_o} \right] \delta x + \left[\left. \frac{\partial F}{\partial u} \right|_{x_o,u_o} \right] \delta u$$

expanding

$$\left. \frac{\partial F}{\partial x} \right|_{x_o,u_o} = Z(x_o)^{-1} \left. \frac{\partial f}{\partial x} \right|_{x_o,u_o} + \left. \frac{\partial Z^{-1}}{\partial x} \right|_{x_o} f(x_o,u_o)$$

$$f(x_o,u_o) = 0$$

$$A = Z(x_o)^{-1} \left. \frac{\partial f}{\partial x} \right|_{x_o,u_o}, \quad B = Z(x_o)^{-1} \left. \frac{\partial f}{\partial u} \right|_{x_o,u_o},$$

$$C = Z(x_o)^{-1} \left. \frac{\partial g}{\partial x} \right|_{x_o,u_o}, \quad D = Z(x_o)^{-1} \left. \frac{\partial g}{\partial u} \right|_{x_o,u_o}.$$

$$\begin{aligned} \dot{x} &= A\delta x + B\delta u \\ y &= C\delta x + D\delta u \end{aligned}$$

Model order reduction

For the linearized model

$$\begin{aligned}\dot{x} &= A\delta x + B\delta u \\ y &= C\delta x + D\delta u\end{aligned}$$

reduced order model can be obtained as

$$\begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} = \begin{bmatrix} \Sigma_\Gamma^{-\frac{1}{2}}(A_{11} - A_{12}A_{22}^\dagger A_{21})\Sigma_\Gamma^{-\frac{1}{2}} & \Sigma_\Gamma^{-\frac{1}{2}}(B_1 - A_{12}A_{22}^\dagger B_2) \\ -(C_1 - C_2 A_{22}^\dagger A_{21})\Sigma_\Gamma^{-\frac{1}{2}} & D_1 - C_2 - A_{22}^\dagger B_2 \end{bmatrix}$$

Reduction process

1. Get controllability & observability Gramians using Lyapunov equation

   $0 = AP + PA^T + BB^T$

   $0 = QA + A^TQ + C^TC.$

2. Choose the desired reduced order n and form the descriptor

   $\Gamma = QP - \zeta^2 I$ where $\sigma_n > \zeta \geq \sigma_{n+1}$

3. Taking singular value decomposition of Γ

   $$\Gamma = [U_{\Gamma 1} \quad U_{\Gamma 2}] \begin{bmatrix} \Sigma_\Gamma & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{\Gamma 1}^T \\ V_{\Gamma 2}^T \end{bmatrix}$$

4. Calculate

   $$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} U_{\Gamma 1}^T \\ U_{\Gamma 2}^T \end{bmatrix} (\zeta^2 A^T + QAP) [V_{\Gamma 1} \quad V_{\Gamma 2}]$$

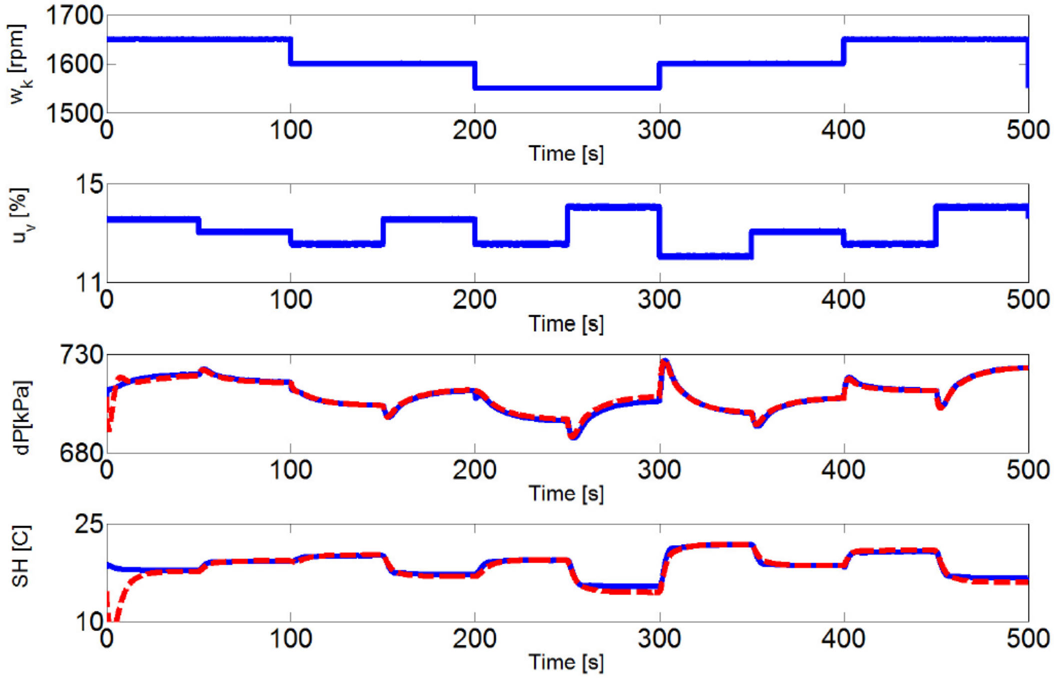
   $$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} U_{\Gamma 1}^T \\ U_{\Gamma 2}^T \end{bmatrix} [QB - C^T]$$

   $$[C_1 \quad C_2] = \begin{bmatrix} CP \\ -\zeta B^T \end{bmatrix} [V_{\Gamma 1} \quad V_{\Gamma 2}]$$

   $$D_1 = D$$

※ $A_{22}^\dagger$ is pseudo inverse of A_{22}

Nonlinear model vs. reduced linear model



Compressor speed(RPM)

EEV opening(%)

Pressure difference(kPa)

Superheat temperature(°C)

< Response of pressure difference and superheat temperature >

Solid line: 4^{th} order linear model

Dotted line: nonlinear model

Fault estimation using PI-observer

Dynamic model including fault and disturbance

$$\begin{aligned} \dot{x}(t) &= A\delta x(t) + B\delta u(t) + B_f f(t) + Gd(t), \\ y(t) &= C\delta x(t), \end{aligned}$$

B_f : actuator fault matrix spanned by matrix *B*,
G : matrix which describe the external disturbance.

Proportional-Integral observer for state and fault detection

$$\begin{aligned} \dot{\hat{x}}(t) &= A\delta\hat{x}(t) + B\delta u(t) + L_P\left(y(t) - \hat{y}(t)\right) + B_f\hat{f}(t), \\ \hat{y}(t) &= C\delta\hat{x}(t), \\ \dot{\hat{f}}(t) &= L_I\left(y(t) - \hat{y}\right), \end{aligned}$$

$\hat{x}$: Estimate of states,
$\hat{y}$: Estimate of fault,
L_P : Proportional observer gain,
L_I : Integral observer gain.

Fault estimation using PI-observer

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$\hat{x}$: Estimate of states,
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Define state & fault estimation error

$$e_x(t) = x(t) - \hat{x}(t), \ e_f(t) = f(t) - \hat{f}(t)$$

Assume $\dot{f}(t) = 0$, then augmented error dynamics is

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_f \end{bmatrix} = \begin{bmatrix} A - L_P C & B_f \\ -L_I C & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_f \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} d$$

Fault estimation using PI-observer

Define $\bar{e}(t) = [\ e_x \quad e_f\ ]$ then

$$\dot{\bar{e}}(t) \quad = \quad \left(\bar{A} - \bar{L}\bar{C}\right)\bar{e}(t) + \bar{G}d(t)$$

where

$$\bar{A} = \begin{bmatrix} A & B_f \\ 0 & 0 \end{bmatrix}, \bar{L} = \begin{bmatrix} L_P \\ L_I \end{bmatrix}, \bar{C} = [\ C \quad 0\ ], \bar{G} = \begin{bmatrix} G \\ 0 \end{bmatrix}.$$

Using H∞ robust theory, robust PI-observer can be designed as

Theorem 1: Given a scalar $\gamma_o > 0$, there exist an H∞ observer if and only if there exists $P = P^T > 0,\ Y$ such that the following matrix inequality

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} - \bar{C}^T Y^T - Y\bar{C} & P\bar{G} & I \\ \bar{G}^T P & -\gamma_o I & 0 \\ I & 0 & -\gamma_o I \end{bmatrix} < 0.$$

The H∞ observer gain matrix is given by $\bar{L} = P^{-1}Y$.

Observer based fault-tolerant control

Fault-tolerant controller design

System Dynamics

$$
\begin{aligned}
\dot{x}(t) &= A\delta x(t) + B\delta u(t) + B_f f(t) + Gd(t), \\
y(t) &= C\delta x(t), \\
\delta u(t) &= K_P \delta\hat{x}(t) + K_f \hat{f}(t)
\end{aligned}
$$

Observer Dynamics

$$
\begin{aligned}
\dot{\hat{x}}(t) &= A\delta\hat{x}(t) + B\delta u(t) + L_P\left(y(t) - \hat{y}(t)\right) + B_f \hat{f}(t), \\
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Fault-tolerant controller design

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$K_f = -B^{\dagger} B_f$

Augmented system dynamics

$$\begin{bmatrix} \dot{x} \\ \dot{\bar{e}} \end{bmatrix} = \begin{bmatrix} (A + BK_P) & \bar{B} \\ 0 & \bar{A} - \bar{L}\bar{C} \end{bmatrix} \begin{bmatrix} \delta x \\ \bar{e} \end{bmatrix} + \begin{bmatrix} G \\ \bar{G} \end{bmatrix} d(t)$$

where $\bar{B} = [0 \quad B_f]$.

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Fault-tolerant controller design

System Dynamics

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where $\bar{B} = [0 \quad B_f]$.

→ This mean that separation property holds, so that the state-feedback gain K_P and the observer gain $\bar{L}$ can be designed separately.

Observer based fault-tolerant control

From system dynamics and input we can get

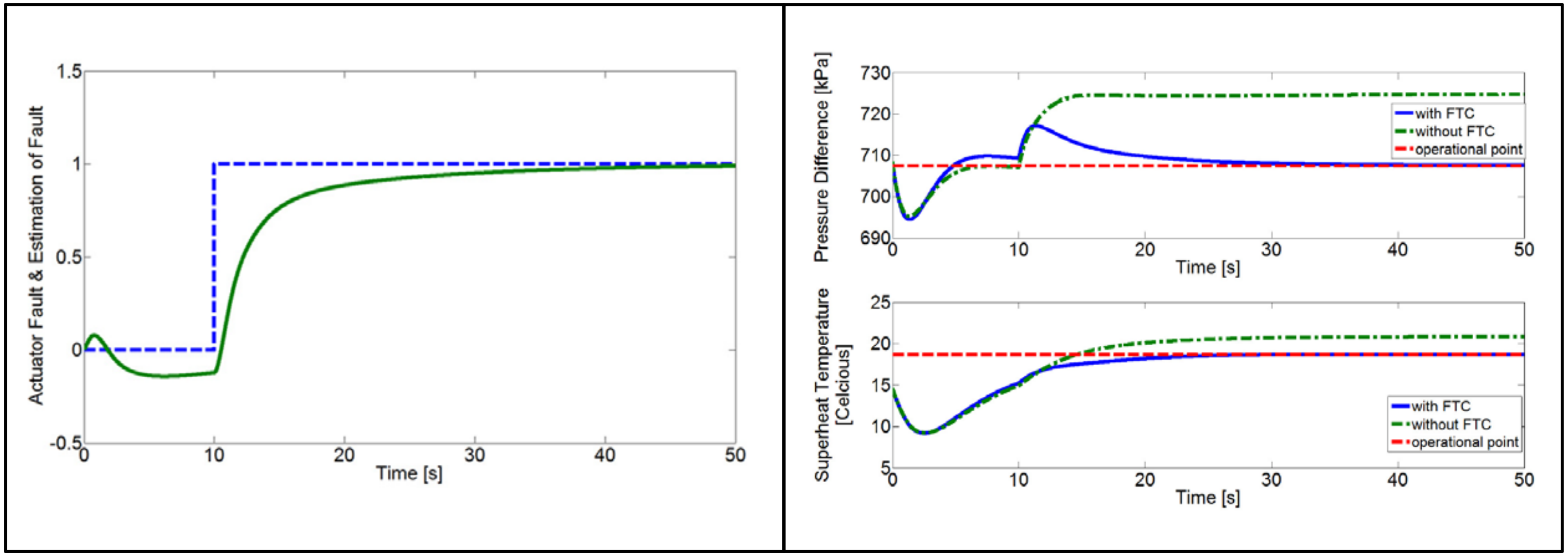
$$\dot{x} = (A + BK_P)\delta x + B_f e_f + Gd$$

Theorem 2: The closed loop VCC system is asymptotically stable and $\|T_{dy}\|_\infty < \gamma_c$ with input constraint $|\delta u_j(t)| \leq \delta u_{j,\max}$ if there exist matrices X>0, $\bar{K}$, and $Z = Z^T$ such that

$$\begin{bmatrix} \Gamma & G & X \\ G^T & -\gamma_c I & 0 \\ X & 0 & -\gamma_c I \end{bmatrix} < 0, \quad \begin{bmatrix} Z & \bar{K} \\ \bar{K}^T & X \end{bmatrix} > 0,$$

where $\Gamma = XA^T + AX + B\bar{K} + \bar{K}^T B^T, Z_{jj} \leq \delta u_{j,\max}^2$ and feedback gain $K_P = \bar{K}X^{-1}$.

Simulation Result



< Compressor fault signal >

< System outputs >

Conclusion

- Reduced linear model of vapor compression cycle system is obtained.
- Robust Fault estimation using PI-observer designed.
- Observer-based Robust Fault-tolerant control with input constraint was proposed.
- Some method can be applied for the system to be stable globally.