Stability and L_2-gain analysis of Impulsive Switched Systems with Average Dwell Time

Application to Hybrid Control

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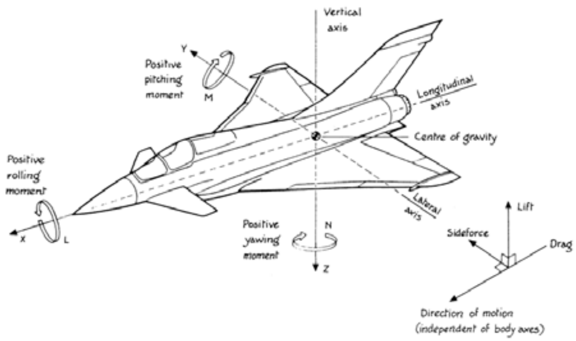
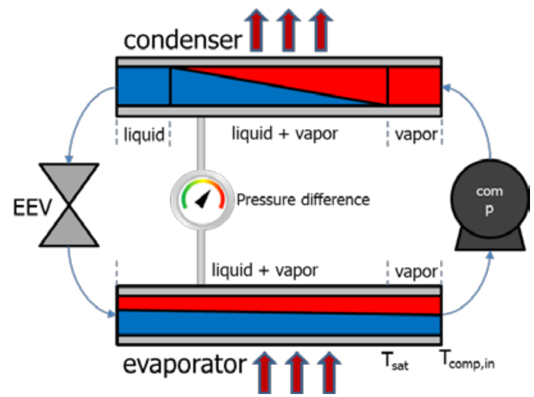
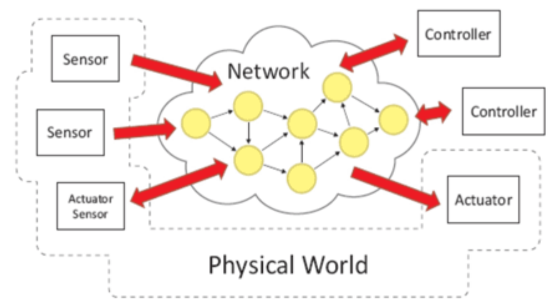


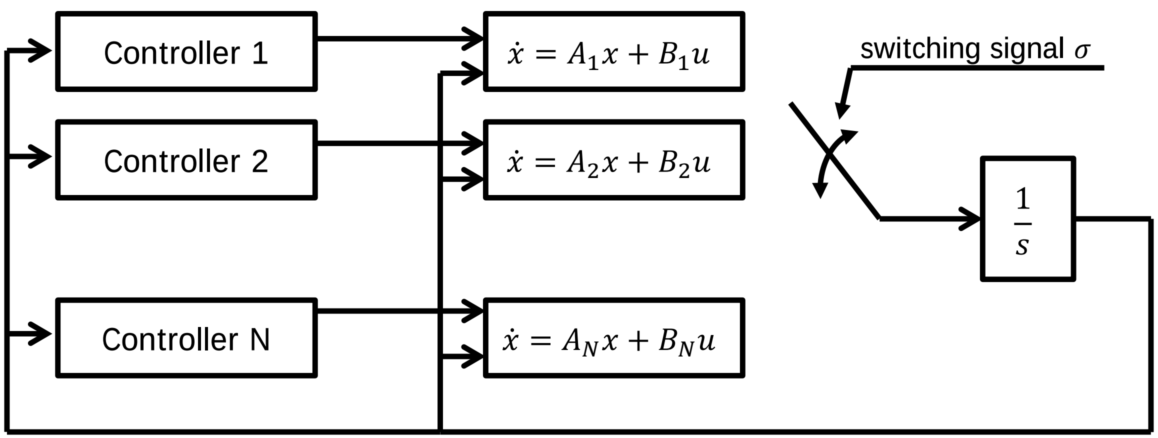
Aircraft



Vapor compression cycle system



Networked control system



Feedback control for switched system

Common Lyapunov function

$V(x) = x^T P x$ Conservative!

Multiple Lyapunov function(MLF) with average dwell time(ADT) switching logic

$$V_1(x) = x^T P_1 x$$
$$V_2(x) = x^T P_2 x$$
$$\vdots$$
$$V_N(x) = x^T P_N x$$
$$V_i \le \mu V_j \quad (i \ne j)$$

Controller	Synthesis condition
$u = K_i x$	LMI
$\dot{x}_c = A_{c,i} x + B_{c,i} y$ $u = C_{c,i} x + D_{c,i} y$	BMI (B. Lu, 2004)

B. Lu, 2006	C. Yuan, 2015	Proposed
$x_c^+ = \Delta_{ij} x$	$x_c^+ = \Delta_{ij} x_c$	$x_c^+ = \Delta_{1,ij} x_c + \Delta_{2,ij} y$

The system suffers additional discrete disturbance

- **Switched Plant**
  $\dot{x} = A_\sigma x + B_\sigma u + E_\sigma w$
  $z = C_{z,\sigma} x + D_{z,\sigma} u + F_{z,\sigma} w$
  $y = C_{y,\sigma} x + D_{y,\sigma} u + F_{y,\sigma} w$

  $\sigma(t)\colon \mathbb{R}_{\geq 0} \to \mathcal{M},\ \mathcal{M} := \{1, \ldots, M\}$
  M: # of subsystems

- **Dynamic controller**
  $\dot{x}_c = A_{c,\sigma} x_c + B_{c,\sigma} y$
  $u = C_{c,\sigma} x_c + D_{c,\sigma} y$

- **Reset Law**
  $x_c^+ = \Delta_{1,ij} x_c + \Delta_{2,ij} y$ at a switching instant

- **Closed-loop dynamics (can be seen as an impulsive switched system)**

$$\dot{\bar{x}} = A_{cl,\sigma}\bar{x} + B_{cl,\sigma} w(t) \qquad t \neq t_k$$
$$z = C_{cl,\sigma}\bar{x} + D_{cl,\sigma} w(t)$$

$$\bar{x}^+ = A_{r,ij}\bar{x} + B_{r,ij} w(t_k),\ t = t_k$$

➡ **Cannot** be analyzed by using the existing weighted L_2-gain

where

$$\bar{x} = [x^T, x_c^T]^T,$$
$$A_{cl,\sigma} = \begin{bmatrix} A_\sigma + B_\sigma D_{c,\sigma} C_{y,\sigma} & B_\sigma C_{c,\sigma} \\ B_{c,\sigma} C_{y,\sigma} & A_{c,\sigma} \end{bmatrix},$$
$$C_{cl,\sigma} = [C_{z,\sigma} + D_{z,\sigma} D_{c,\sigma} C_{z,\sigma} \quad D_z C_{c,\sigma}],$$
$$B_{cl,\sigma} = \begin{bmatrix} E_\sigma + B_\sigma D_{c,\sigma} F_{y,\sigma} \\ B_{c,\sigma} F_{y,\sigma} \end{bmatrix}, D_{cl,\sigma} = F_{z,\sigma} + D_{z,\sigma} D_{c,\sigma} F_{y,\sigma},$$
$$A_{r,ij} = \begin{bmatrix} I & 0 \\ \Delta_{\mathbf{2},\boldsymbol{ij}} C_{y,i} & \Delta_{1,ij} \end{bmatrix}, B_{r,ij} = \begin{bmatrix} 0 \\ \Delta_{2,ij} F_{y,i} \end{bmatrix}.$$

- Weighted L_2-gain

$$\int_0^\infty e^{-\lambda_0 t} z^T(t) z(t)\,dt \leq \gamma^2 \int_0^\infty w^T(t) w(t)\,dt$$

Question to be answered

1. How to take account both continuous and discrete disturbance together into L_2-gain performance?

2. What is the condition that guarantees stability and L_2-gain performance of the impulsive switched system (or the closed loop system)?

3. How to design the hybrid controller that has output information on reset law?

Weighted mixed L_2/l_2-gain

- Definition(weighted mixed L_2/l_2-gain): The impulsive switched system has a weighted mixed L_2/l_2-gain from input w to output z smaller than or equal to γ if the system is asymptotically stable and satisfies following inequality:

$$\int_0^\infty e^{-\lambda_0 t} z^T(t)z(t)dt \leq \gamma^2\left\{\int_0^\infty w^T(t)w(t)dt + \sum_{k=1}^\infty w^T(t_k)w(t_k)\right\}$$

$\lambda_0 > 0$: decay rate
t_k: switching instants

Stability and L$_2$-gain performance

- *Theorem 1*: The impulsive system is globally uniformly asymptotically stable(GUAS) for every switching signal σ with average dwell time $\tau_a \geq \ln(\mu)/\lambda_0$ and achieves a weighed mixed L_2/l_2-gain under zero initial condition if, for given scalars $\gamma > 0, \mu \geq 1, \lambda_0 > 0$ and ε, there exist matrices $P_i > 0, Q_i$ with appropriate dimensions satisfying the following matrix inequalities $\forall i, j \in \mathcal{M} \times \mathcal{M},\ i \neq j$:

$$\begin{bmatrix} Q_i^T A_{cl,i} + A_{cl,i}^T Q_i + \lambda_0 P_i & \star & \star & \star \\ P_i - Q_i + \varepsilon Q_i^T A_{cl,i} & -\varepsilon\left(Q_i + Q_i^T\right) & \star & \star \\ B_{cl,i}^T Q_i & \varepsilon B_{cl,i}^T Q_i & -\gamma^2 I & \star \\ C_{cl,i} & 0 & D_{cl,i} & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} P_j - Q_j - Q_j^T & \star & \star \\ A_{r,ij}^T Q_j & -\mu P_i & \star \\ B_{r,ij}^T Q_j & 0 & -\gamma^2 I \end{bmatrix} \leq 0.$$

Question to be answered

- ✓ How to take account both continuous and discrete disturbance together into L_2-gain performance?
- ✓ What is the condition that guarantees stability and L_2-gain performance of the impulsive switched system (or the closed loop system)?

3. How to design the hybrid controller that has output information on reset law?

Design algorithm of Hybrid control

- *Theorem 2:* The closed-loop system is GUAS for every switching signal σ with average dwell time $\tau_a \geq \ln(\mu)/\lambda_0$ and achieves a weighed mixed L_2/l_2-gain under zero initial condition if, for given scalars $\gamma > 0, \mu \geq 1, \lambda_0 > 0$ and ε, there exist matrices $P_i > 0, X_i, Y_i, W_i, \hat{A}_{c,i}, \hat{B}_{c,i}, \hat{C}_{c,i}, \hat{D}_{c,i}, \hat{\Delta}_{1,ij}, \hat{\Delta}_{2,ij}$ with appropriate dimensions satisfying the following matrix inequalities $\forall i,j \in \mathcal{M} \times \mathcal{M},\ i \neq j$:

$$\begin{bmatrix} \Phi_{11,i} + \Phi_{11,i}^T + \lambda_0 \hat{P}_i & \star & \star & \star \\ \hat{P}_i + \varepsilon\Phi_{11,i} - \Phi_{22,i}^T & -\varepsilon(\Phi_{22,i} + \Phi_{22,i}^T) & \star & \star \\ \Phi_{31,i} & \varepsilon\Phi_{31,i} & -\gamma^2 I & \star \\ \Phi_{41,i} & 0 & F_{z,i} + D_{z,i}\hat{D}_{c,i}F_{y,i} & -I \end{bmatrix} < 0, \quad \begin{bmatrix} \hat{P}_j - \Phi_{22,j} - \Phi_{22,j}^T & \star & \star \\ \Theta_{21,ij}^T & -\mu\hat{P}_i & \star \\ [0 \quad F_{y,i}^T\hat{\Delta}_{1,ij}^T] & 0 & \gamma^2 I \end{bmatrix} \leq 0.$$

where

$$\Phi_{11,i} = \begin{bmatrix} A_i X_i + B_i \hat{C}_{c,i} & A_i + B_i \hat{D}_{c,i} C_{y,i} \\ \hat{A}_{c,i} & Y_i A_i + \hat{B}_{c,i} C_{y,i} \end{bmatrix}, \Phi_{22,i} = \begin{bmatrix} X_i & I \\ W_i & Y_i \end{bmatrix}, \quad \Phi_{31,i} = [E_i^T + F_{y,i}^T \hat{D}_{c,i}^T B_i^T \quad E_i^T Y_i^T + F_{y,i}^T \hat{B}_{c,i}],$$

$$\Phi_{41,i} = [C_{z,i} X_i + F_{z,i}\hat{C}_{c,i} \quad C_{z,i} + F_{z,i}\hat{D}_{c,i} C_{y,i}], \ \Theta_{21,ij} = \begin{bmatrix} X_i & I \\ \hat{\Delta}_{2,ij} & Y_j + \hat{\Delta}_{1,ij} C_{y,i} \end{bmatrix}.$$

- Designing algorithm of Hybrid controller

Step 1: Obtain matrix variables by solving LMIs.
Step 2: Construct M_i and N_i from the relation that $M_i N_i^T = W_i - X_i Y_i$
Step 3: Obtain the hybrid controller gains as follows:

$$\begin{bmatrix} A_{c,i} & B_{c,i} \\ C_{c,i} & D_{c,i} \end{bmatrix} = \begin{bmatrix} N_i & Y_i B_i \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} \hat{A}_{c,i} - Y_i A_i X_i & \hat{B}_{c,i} \\ \hat{C}_{c,i} & \hat{D}_{c,i} \end{bmatrix} \begin{bmatrix} M_i^T & 0 \\ C_{y,i} & I \end{bmatrix}^{-1}$$

$$\Delta_{1,ij} = N_j^{-1}\hat{\Delta}_{1,ij},$$

$$\Delta_{2,ij} = N_j^{-1}(\hat{\Delta}_{2,ij} - Y_j X_i - \hat{\Delta}_{1,ij} C_{y,i} X_i) M_i^{-T}.$$

- Optimization problem

$$\min \ \gamma^2$$

subject to the LMI conditions

Compared result

System parameters

Subsystem 1)

$$A_1 = \begin{bmatrix} 0.5108 & -0.9147 & -0.2 \\ -0.6563 & 0.1798 & 0.113 \\ 0.881 & -0.7841 & 0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.3257 \\ 1.2963 \\ 2.43 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1056 \\ 0.1284 \\ 0.1 \end{bmatrix},$$
$$C_{z,1} = [0.01 \quad 0.06 \quad 0.03], C_{y,1} = [-5 \quad 0.2 \quad 0.5],$$
$$D_{z,1} = F_{z,1} = 0, F_{y,1} = 0.1.$$

Subsystem 2)

$$A_2 = \begin{bmatrix} -0.125 & -0.9833 & -0.34 \\ -0.5305 & 0.3848 & 0.58 \\ 1.0306 & 0.6521 & 0.1 \end{bmatrix}, B_2 = \begin{bmatrix} 1.0992 \\ 0.6532 \\ 3.5 \end{bmatrix}, E_2 = \begin{bmatrix} 0.7425 \\ 0.1436 \\ 0.1 \end{bmatrix},$$
$$C_{z,2} = [0.01 \quad 0.02 \quad 0.05], C_{y,2} = [-6 \quad 6 \quad -1],$$
$$D_{z,2} = F_{z,2} = 0, F_{y,2} = 0.1.$$

Given parameters $\lambda_0 = 0.12$ and $\varepsilon = 0.1$, we obtained $\gamma_{\min}$ solving the optimization problem for various μ.

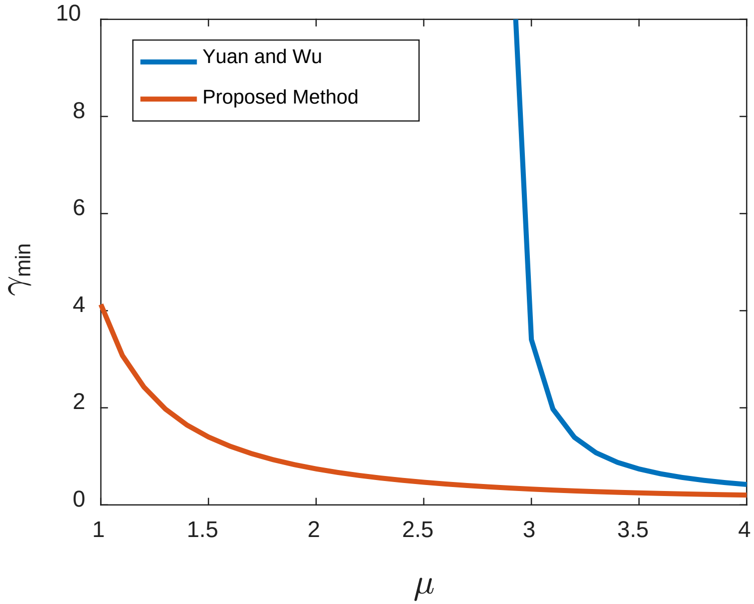


Table 1

$\gamma_{\min}$ in example ((!) means infeasible)

μ	1.0001	2.0	3.0	4.0
ADT bound $\ln(\mu)/\lambda_0$	**0.00083**	**5.7762**	**9.1551**	**11.5525**
Yuan and Wu	(!)	(!)	3.4044	0.4210
Theorem 2	**4.1289**	**0.7418**	**0.3252**	**0.2023**

90.4% reduced **52% reduced**

Almost arbitrary switching is almost achieved

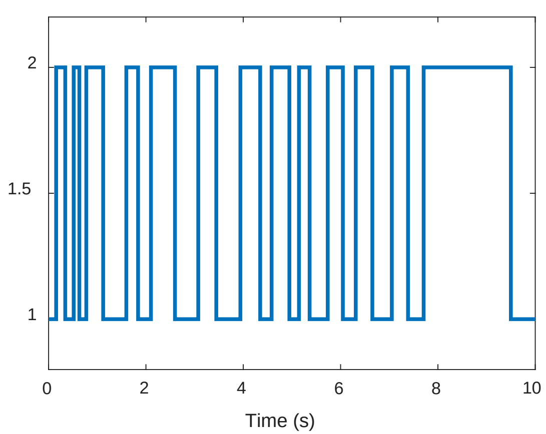
Time-domain simulation

- Using Theorem 2 with parameters $\mu = 1.001, \varepsilon = 0.1, \gamma = 4.2$ and $\lambda_0 = 0.12$, hybrid controller is designed as

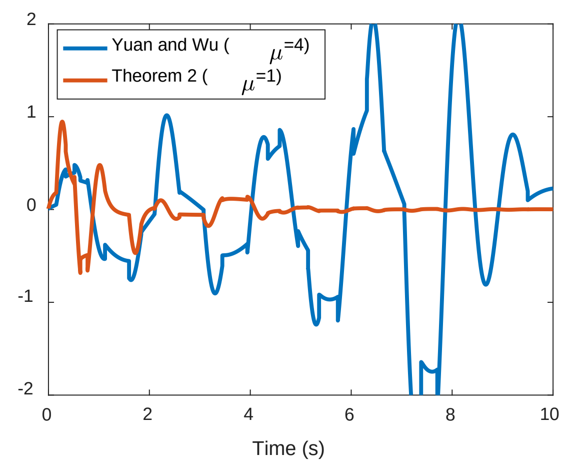
$$\left[\begin{array}{c|c} A_{c,1} & B_{c,1} \\ \hline C_{c,1} & D_{c,1} \end{array}\right] = \left[\begin{array}{ccc|c} -7.9079 & 0.6270 & -1.3680 & 0.6248 \\ 1.5209 & -0.4055 & 8.2508 & -0.1240 \\ 33.8657 & -8.6707 & -212.4614 & -2.7498 \\ \hline 24.6294 & -2.4577 & 6.0935 & -2.0481 \end{array}\right] \quad \left[\begin{array}{c|c} A_{c,2} & B_{c,2} \\ \hline C_{c,2} & D_{c,2} \end{array}\right] = \left[\begin{array}{ccc|c} 15.9078 & -102.1759 & -3.2489 & 1.7283 \\ 8.7176 & -53.3806 & 3.5117 & 0.9012 \\ -18.9715 & 111.3551 & -189.5649 & -1.9520 \\ \hline -47.7589 & 293.0891 & 9.4324 & -4.9321 \end{array}\right]$$

$$\Delta_{1,12} = \begin{bmatrix} 0,0037 \\ -0.0516 \\ -0.0118 \end{bmatrix}, \Delta_{2,12} = \begin{bmatrix} 0.9596 & -0.0876 & 0.0009 \\ 0.6349 & 0.4371 & 0.0196 \\ 0.1454 & -0.0373 & -0.0302 \end{bmatrix} \quad \Delta_{2,12} = \begin{bmatrix} 0.0037 \\ 0.0656 \\ -0.0113 \end{bmatrix}, \Delta_{2,12} = \begin{bmatrix} 1.0313 & -0.0473 & -0.0011 \\ 0.6366 & -2.0621 & -0.0582 \\ -0.1101 & 0.6489 & -0.0182 \end{bmatrix}$$

- Simulation conditions: $x(0) = [1,0,0]^T, w(t) = 0.1\sin(0.3t)$



Switching signal with average dwell time $\tau_a = \frac{10}{26} = 0.3836$



Trajectory of the controlled output

Summary

1. How to take account continuous and discrete disturbance together into L_2-gain analysis?
   - We introduced weighted mixed L_2/l_2-gain

2. What is the condition that guarantees stability and L_2-gain of the impulsive switched system (closed loop system)?
   - Theorem 1 is the answer for this

3. How to design the hybrid controller that has output information on reset law?
   - Theorem 2 is the answer for this

Thank you for listening

Q&A