Generalized complex projective synchronization of chaotic complex systems with unknown parameters

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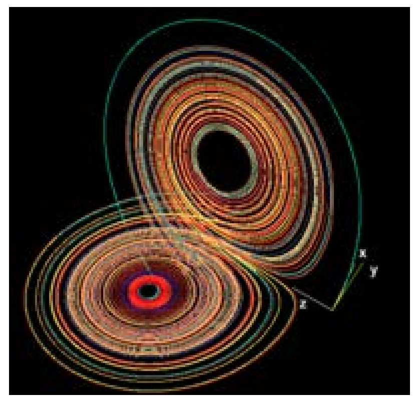
Presentation Outline

- Chaotic system
  - Synchronization
  - Complex number
- Problem Formulation
- Adaptive synchronization
  - Theorem
  - Proof of Stability
- Result
  - Numerical example
  - Simulation result
- Conclusion

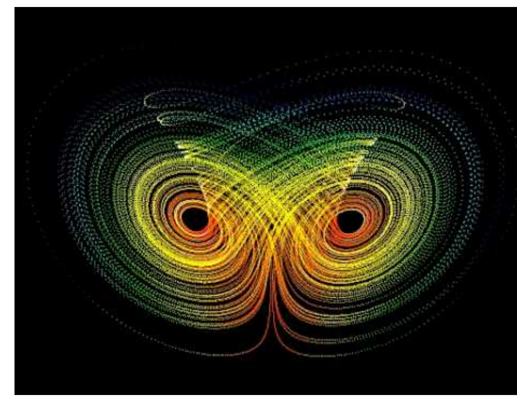
Small difference in initial conditions yield widely diverging outcomes

Examples of Chaotic system

Weather system



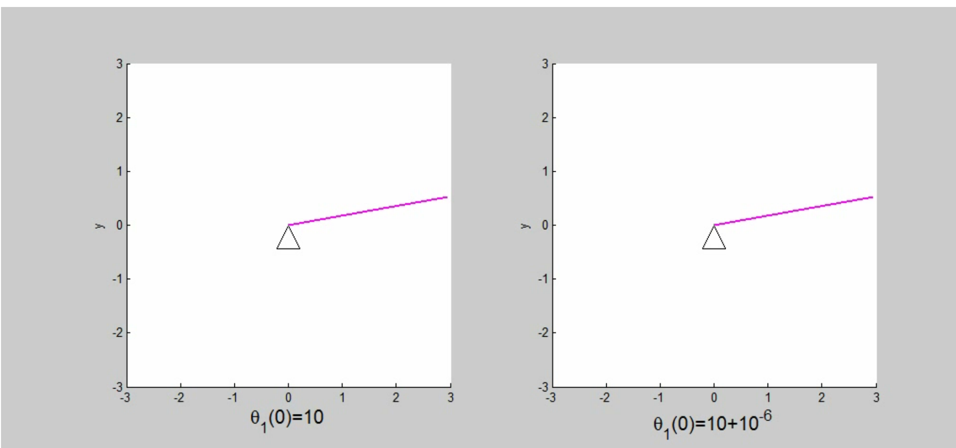
Lorenz attractor

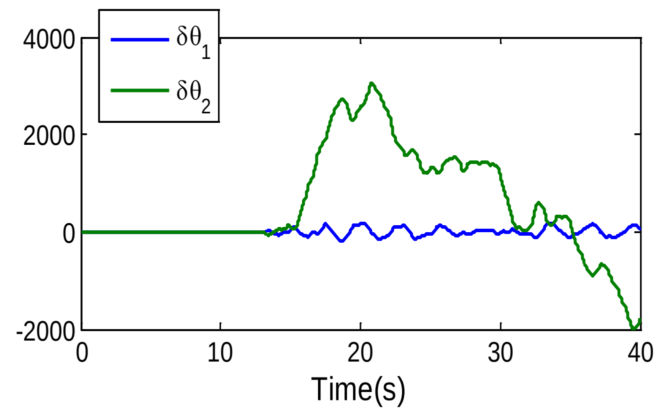


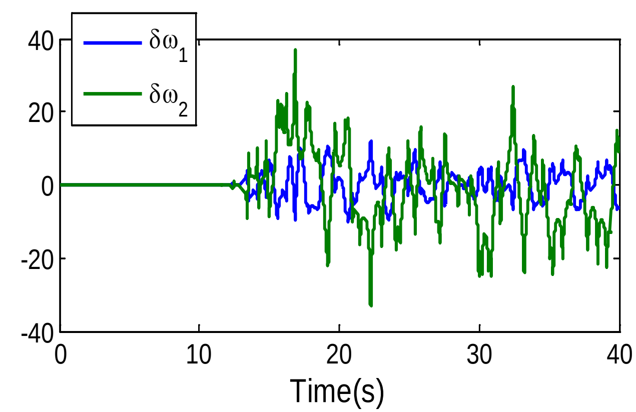
Chen attractor

Small difference in initial conditions yield widely diverging outcomes

Chaotic behavior of double rod pendulum







Initial conditions: $\theta(0) = 10$ $\theta(0) = 10 + 10^{-6}$

Synchronization is to make states of two systems same

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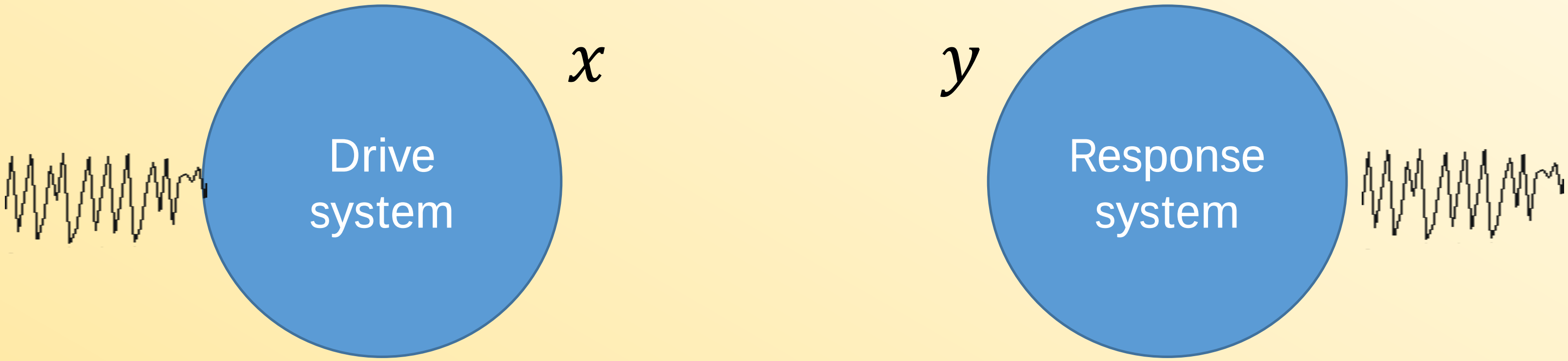
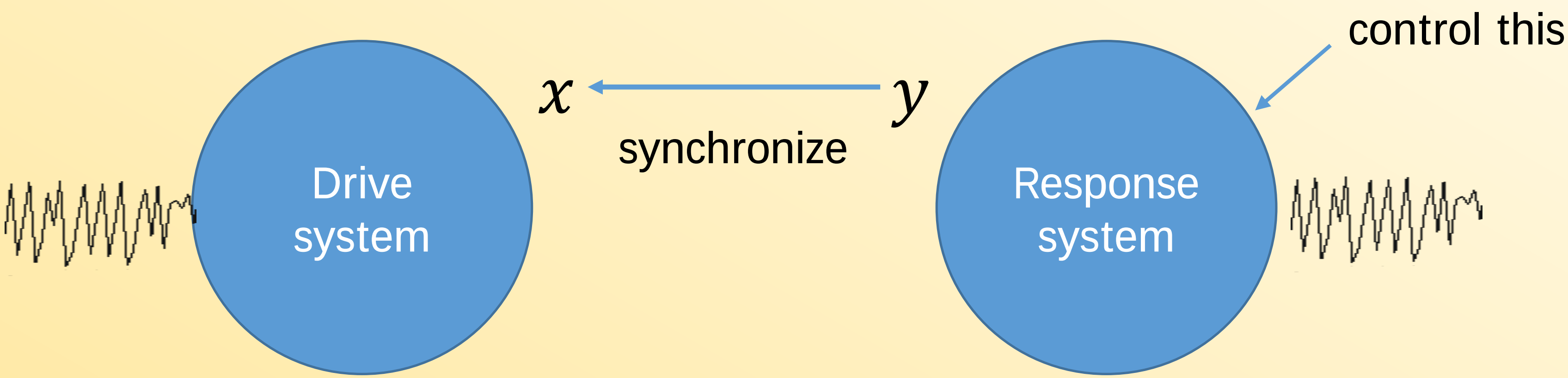


Synchronization is to make states of two systems same



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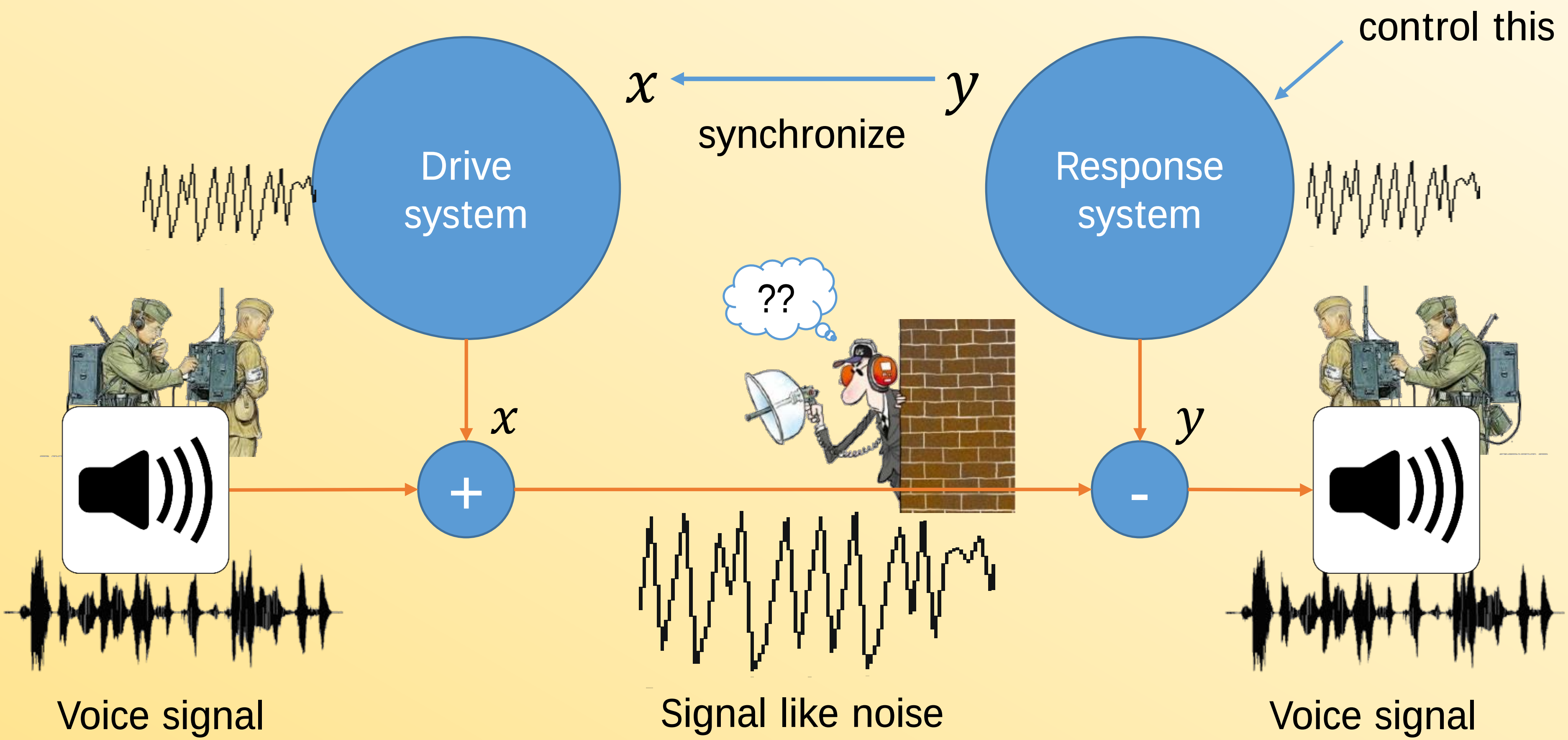

What we did is..

- Synchronization of chaotic system with:
  - Complex number states

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  - Complex number states
  - Different dimensional

What we did is..

- Synchronization of chaotic system with:
  - Complex number states
  - Different dimensional
  - Unknown parameter

Problem Formulation

- Consider the following drive-response chaotic complex systems

$$\dot{\mathbf{x}} = f(\mathbf{x}) + F(\mathbf{x})\theta \qquad (1)$$

$$\dot{\mathbf{y}} = g(\mathbf{y}) + G(\mathbf{y})\varphi + \mathbf{u}(t), \qquad (2)$$

$\mathbf{x}(t) = [x_1, \cdots, x_m]^T$ is a state complex vector of the drive system (1),

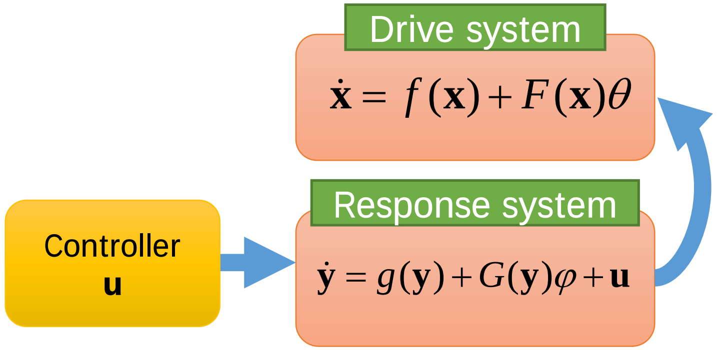
$\mathbf{y}(t) = [y_1, \cdots, y_n]^T$ is a state complex vector of the response system (2),

- Define the error vector

$$\mathbf{e}(t) = \mathbf{y}(t) - H\mathbf{x}(t), \qquad (3)$$

$H \in C^{n \times m}$ is a nonzero complex projective matrix whose components are constant complex numbers.

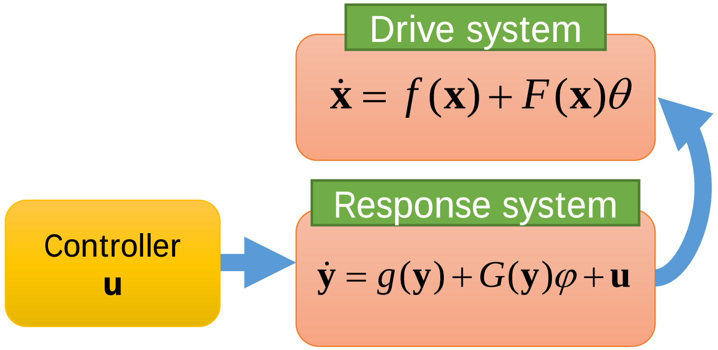
Method



Make **y** to follow **x**

Subscript ‘r’ and ‘i’ represent real parts and imaginary parts of a complex matrix or vector or variable.

Method



Make **y** to follow **x**

1. Use scaling matrix to match dimensions

$$e(t) = y(t) - Hx(t),$$

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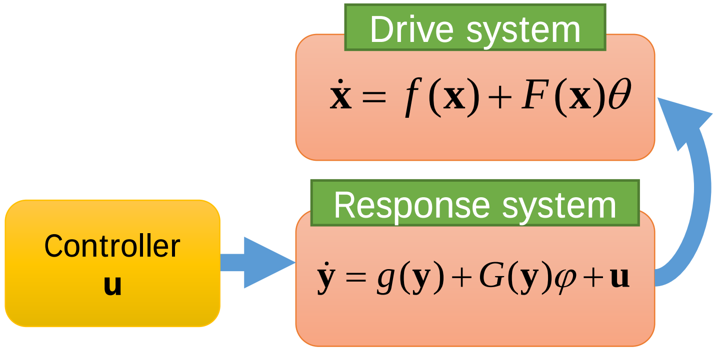
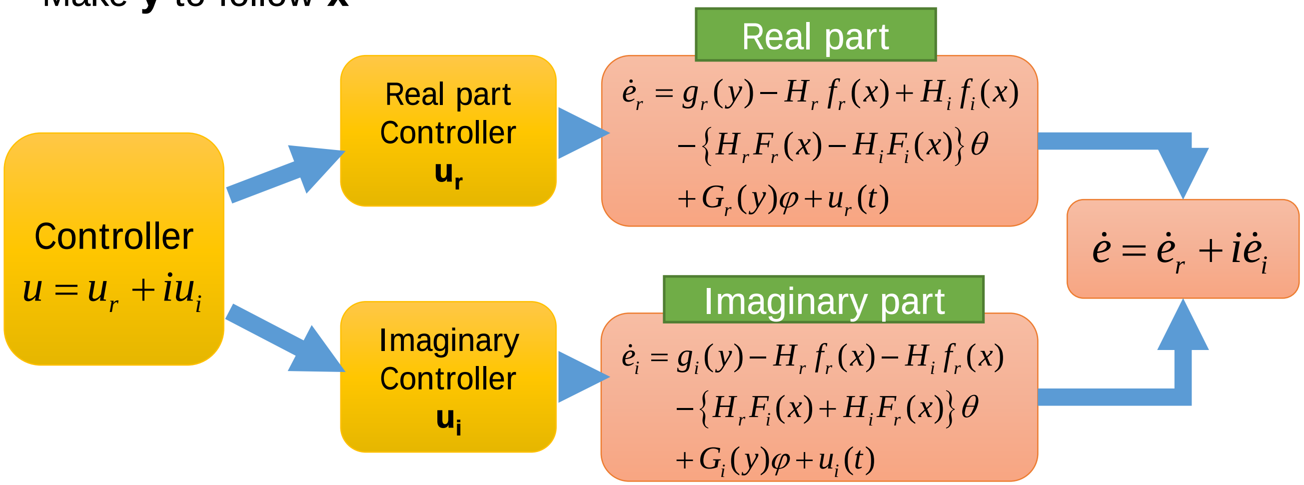


Make **y** to follow **x**

1. Use scaling matrix to match dimensions

$$e(t) = y(t) - Hx(t),$$

2. Derive error dynamics separately & Design controllers separately into real and imaginary



Subscript ‘r’ and ‘i’ represent real parts and imaginary parts of a complex matrix or vector or variable.

Control inputs and Update laws for an adaptive controller

Theorem 1: *For the chaotic complex drive system (1), the response system (2) and the scaling matrix H, the Adaptive generalized complex projective synchronization(AGCPS) of the chaotic complex systems is accomplished if a control input and adaptive laws are chosen as follows*

- Control inputs

$$\begin{aligned} u_r(t) &= -g_r(y) + H_r F_r(x) - H_i F_i(x) \\ &\quad + \{H_r F_r(x) - H_i F_i(x)\}\hat{\theta} \\ &\quad - G_r(y)\hat{\varphi} - K e_r(t) \end{aligned} \quad (4)$$

$$\begin{aligned} u_i(t) &= -g_i(y) + H_r F_i(x) + H_i F_r(x) \\ &\quad + \{H_r F_i(x) + H_i F_r(x)\}\hat{\theta} \\ &\quad - G_i(y)\hat{\varphi} - K e_i(t) \end{aligned} \quad (5)$$

- Update Laws

$$\begin{aligned} \dot{\hat{\theta}} &= -K_\theta \Big[\{H_r F_r(x) - H_i F_i(x)\}^T e_r(t) \\ &\quad + \{H_r F_i(x) + H_i F_r(x)\}^T e_i(t) \Big] \end{aligned} \quad (6)$$

$$\dot{\hat{\varphi}} = K_\phi \left[G_r(y)^T e_r(t) + G_i(y)^T e_i(t) \right] \quad (7)$$

'hat' represents the estimation of parameter vector of a complex chaotic systems.

Proof of the theorem

- Choose the following Lyapunov function candidate:

$$V(t)=\frac{1}{2}\left\{\mathbf{e}_r{}^T\mathbf{e}_r+\mathbf{e}_i{}^T\mathbf{e}_i\right\}+\frac{1}{2}K_\theta{}^{-1}\tilde{\theta}^T\tilde{\theta}+\frac{1}{2}K_\phi{}^{-1}\tilde{\phi}^T\tilde{\phi}, \tag{8}$$

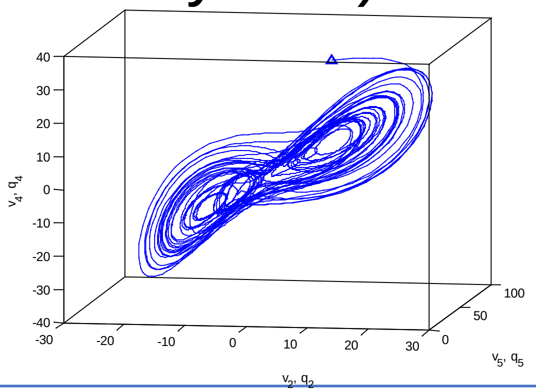
where $\tilde{\theta}=\hat{\theta}-\theta,$ and $\tilde{\phi}=\hat{\phi}-\phi.$

- Take a time derivative of (8) and substituting (4)~(7) yields

$$\begin{aligned}\dot{V}(t)=\mathbf{e}_r{}^T&\left[\left\{H_rF_r(\mathbf{x})-H_iF_i(\mathbf{x})\right\}\tilde{\theta}-G_r(\mathbf{y})\tilde{\phi}-K\mathbf{e}_r(t)\right]\\ &+\mathbf{e}_i{}^T\left[-\left\{H_rF_i(\mathbf{x})+H_iF_r(\mathbf{x})\right\}\tilde{\theta}-G_i(\mathbf{y})\tilde{\phi}-K\mathbf{e}_i(t)\right]\\ &+K_\theta{}^{-1}\tilde{\theta}^T\dot{\hat{\theta}}+K_\phi{}^{-1}\tilde{\phi}^T\dot{\hat{\phi}}.\\ =&-\mathbf{e}_r{}^TK\mathbf{e}_r(t)-\mathbf{e}_i{}^TK\mathbf{e}_i(t)\\ =&-\sum_{j=1}^{n}k_j\left[e_{rj}{}^2+e_{ij}{}^2\right]\leq k_{\min}\left\|\mathbf{e}(t)\right\|\end{aligned}$$

Numerical Simulations

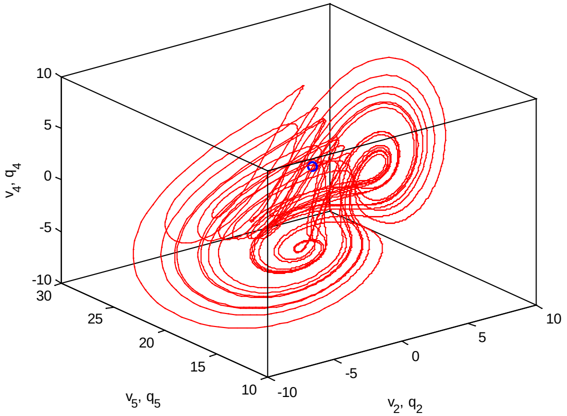
- Hyperchaotic complex Lorenz-type system (Drive system)



$$\begin{cases} \dot{x}_1 = \alpha\left(x_2 - x_1\right) \\ \dot{x}_2 = \gamma x_1 - x_2 - x_1 x_3 + x_4 \\ \dot{x}_3 = \frac{1}{2}\left(\overline{x}_1 x_2 + x_1 \overline{x}_2\right) - \beta x_3 \\ \dot{x}_4 = \rho x_1 + \mu x_2 \end{cases}$$

$\alpha = 14, \beta = 3, \gamma = 45, \rho = -5, \mu = -4$

- Complex Chen system (Response System)



$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + u_1 \\ \dot{y}_2 = \left(b - a\right) y_1 + b y_2 - y_1 y_3 + u_2 \\ \dot{y}_3 = \frac{1}{2}\left(\overline{y}_1 y_2 + y_1 \overline{y}_2\right) - c y_3 + u_3 \end{cases}$$

$a = 28, b = 22, c = 1$

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- Complex projective matrix

$$H = \begin{bmatrix} 0.1 + 0.5i & 0.1i & 0 & 0 \\ 0 & 0.5 + 0.2i & 0.5 + 0.1i & 0 \\ 0 & 0 & 0.1 & 0.2 + 0.5i \end{bmatrix}$$

Numerical Simulations

- We can represent the drive system and the response system as a form of (1) and (2), then

  - Drive system

$$\dot{\mathbf{x}} = f_r(\mathbf{x}) + F_r(\mathbf{x})\theta + i\{ f_i(\mathbf{x}) + F_i(\mathbf{x})\theta \},$$

$$f_r(\mathbf{x}) = \begin{bmatrix} 0 \\ -x_{r1}x_{r3} - x_{r3} + x_{r4} \\ x_{r1}x_{r2} + x_{i1}x_{i2} \\ 0 \end{bmatrix}, \quad f_i(\mathbf{x}) = \begin{bmatrix} 0 \\ -x_{i1}x_{r3} - x_{i2} + x_{i4} \\ 0 \\ 0 \end{bmatrix},$$

$$F_r(\mathbf{x}) = \begin{bmatrix} x_{r2} - x_{r1} & 0 & 0 & 0 & 0 \\ 0 & 0 & x_{r1} & 0 & 0 \\ 0 & -x_{r3} & 0 & 0 & 0 \\ 0 & 0 & 0 & x_{r1} & x_{r2} \end{bmatrix},$$

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$$\theta = [\alpha \quad \beta \quad \gamma \quad \rho \quad \mu]^T$$

  - Response system

$$\dot{\mathbf{y}} = g_r(\mathbf{y}) + G_r(\mathbf{y})\varphi + i\{ g_i(\mathbf{y}) + G_i(\mathbf{y})\varphi \} + \mathbf{u}(t),$$

$$g_r(\mathbf{x}) = \begin{bmatrix} 0 \\ -y_{r1}y_{r3} \\ y_{r1}y_{r3} + y_{i1}y_{i2} \end{bmatrix}, \; g_i(\mathbf{x}) = \begin{bmatrix} 0 \\ -y_{i1}y_{i3} \\ 0 \end{bmatrix},$$

$$G_r(\mathbf{x}) = \begin{bmatrix} y_{r2} - y_{r1} & 0 & 0 \\ -y_{r1} & y_{r1} + y_{r2} & 0 \\ 0 & 0 & -y_{r3} \end{bmatrix},$$

$$G_i(\mathbf{x}) = \begin{bmatrix} y_{i2} - y_{i1} & 0 & 0 \\ -y_{i1} & y_{i1} + y_{i2} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\phi = [a \quad b \quad c]^T$$

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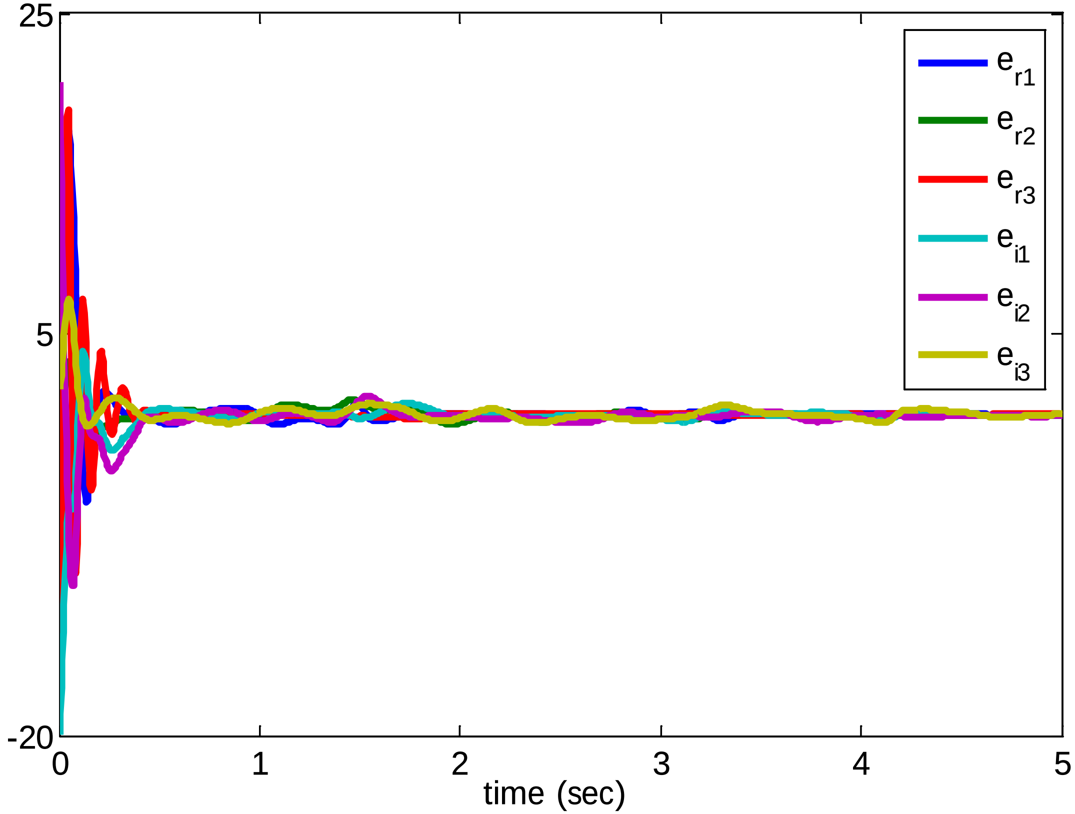
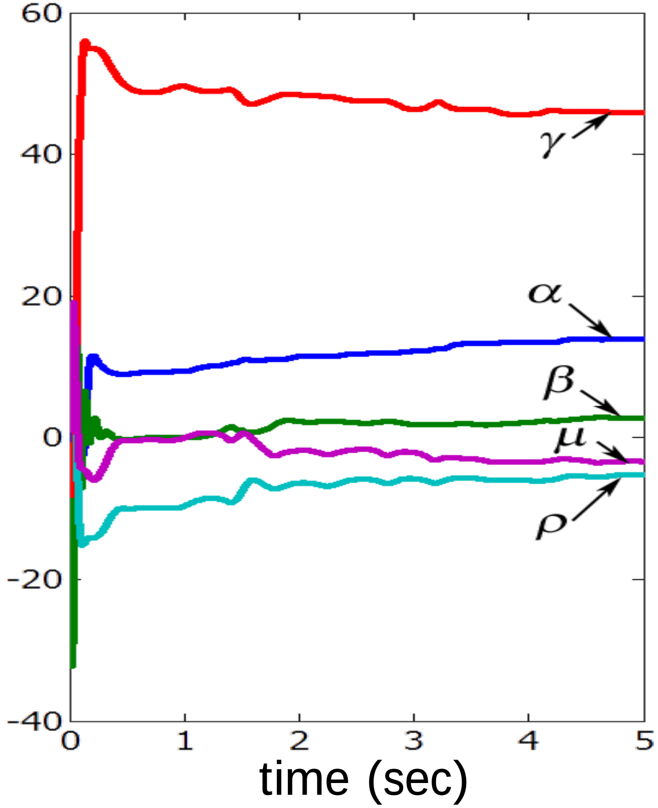
States are synchronized by using adaptive controller

- Synchronization errors in an example

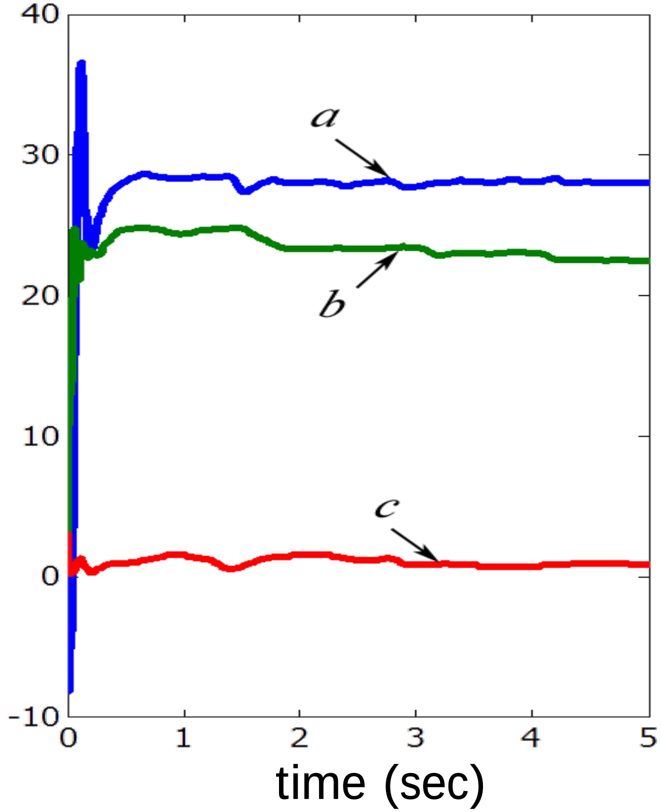


Parameters converge to actual values using adaptive law

< Parameters of Drive system>



< Parameters of Response system >



Conclusions:

- With complex number, chaotic systems becomes much more complicated.
- Complex number chaotic systems can be synchronized although the parameters of the systems are unknown.
- Using scaling matrix, synchronizations of chaotic systems which have different dimension can be accomplished.