Fault estimation and Fault-tolerant control of vapor compression cycle systems

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What is VCC systems?

< Vapor Compression Cycle System >

< Vapor Compression Cycle >
What is VCC systems?

< Vapor Compression Cycle System >

< Vapor Compression Cycle >

applications
Pressure and superheat temperature are important things to be controlled.

Superheat temperature: temperature of a vapor above the saturated vapor temperature.
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Superheat temperature: temperature of a vapor above the saturated vapor temperature.
Pressure and superheat temperature are important things to be controlled.

Actuators
- Compressor Speed (RPM)
- Electrical Expansion Valve (%)

Controlled output
- Pressure difference between condenser and evaporator (kPa)
- Superheat temperature of an evaporator (K)

Superheat temperature: temperature of a vapor above the saturated vapor temperature.
Pressure and superheat temperature are important thing to be controlled.

Superheat temperature: temperature of a vapor above the saturated vapor temperature ($T_{\text{comp,in}} - T_{\text{sat}}$).
Fault cause severe damage to the system

Superheat temperature: temperature of a vapor above the saturated vapor temperature ($T_{\text{comp, in}} - T_{\text{sat}}$).
Nonlinear model for VCC system

Static Models
- Compressor model
  \[
  \dot{m}_k = \rho_k V_k \omega_k \eta_{vol}, \\
  h_{out} = \frac{1}{\eta_k} [h_{out,\text{isentropic}} + h_{in}] (\eta_k - 1),
  \]
- Electrical expansion valve
  \[
  \dot{m}_v = C_d \sqrt{\rho_v (P_{in} - P_{out})}, \\
  h_{v,\text{in}} = h_{v,\text{out}},
  \]

Dynamic models
- Condenser
  \[
  Z_c(x_c) \dot{x}_c = f_c(x_c, u) \\
  x_c = [L_{c1} \ L_{c2} \ P_c \ h_{out} \ T_{wc1} \ T_{wc2} \ T_{wc3}]^T \\
  u = [w_k \ u_v \ m_{ea} \ m_{ea} \ T_{ea,\text{in}} \ T_{ea,\text{in}}]^T
  \]
- Evaporator
  \[
  Z_e(x_e) \dot{x}_e = f_e(x_e, u) \\
  x_e = [L_{e1} \ P_e \ h_{out} \ T_{we1} \ T_{we2}]^T \\
  u = [w_k \ u_v \ m_{ea} \ m_{ea} \ T_{ea,\text{in}} \ T_{ea,\text{in}}]^T
  \]
Nonlinear model for VCC system

Static Models

- Compressor model
  \[ \dot{m}_k = \rho_k V_k w_k \eta_{vol}, \]
  \[ h_{out} = \frac{1}{\eta_k} [h_{out, isentropic} + h_{in}] (\eta_k - 1), \]

- Electrical expansion valve
  \[ \dot{m}_v = C_d \sqrt{\rho_v (P_{in} - P_{out})}, \]
  \[ h_{v, in} = h_{v, out}, \]

Dynamic models

- Condenser
  \[ Z_c(x_c) \dot{x}_c = f_c(x_c, u) \]
  \[ x_c = [L_{c1} L_{c2} P_c h_{c1} h_{c2} T_{wc1} T_{wc2} T_{wc3}]^T \]
  \[ u = [w_k u_v \dot{m}_{ea} \dot{m}_{ca} T_{ea, in} T_{ca, in}]^T \]

- Evaporator
  \[ Z_e(x_e) \dot{x}_e = f_e(x_e, u) \]
  \[ x_e = [L_{e1} P_e h_{eout} T_{e1} T_{e2}]^T \]
  \[ u = [w_k u_v \dot{m}_{ea} \dot{m}_{ca} T_{ea, in} T_{ca, in}]^T \]

- Nonlinear descriptor form
  \[ \begin{bmatrix} Z_c(x_c) & 0_{7 \times 5} \\ 0_{5 \times 7} & Z_e(x_e) \end{bmatrix} \begin{bmatrix} \dot{x}_c \\ \dot{x}_e \end{bmatrix} = \begin{bmatrix} f_c(x_c, u) \\ f_e(x_e, u) \end{bmatrix}, \]
  \[ y = g(x_c, x_e) = [\Delta P \quad SH]^T, \quad u = [w_k \quad u_v \quad \dot{m}_{ea} \quad \dot{m}_{ca} \quad T_{ea, in} \quad T_{ca, in}]^T. \]
Linearization of Nonlinear model

Given nonlinear model

\[
\begin{align*}
Z(x, u) \dot{x} &= f(x, u) \\
y &= g(x)
\end{align*}
\]

\[
\begin{align*}
Z(x, u) \dot{x} &= f(x, u) \\
\dot{x} &= Z(x, u)^{-1} f(x, u) \\
&= F(x, u).
\end{align*}
\]
Linearization of Nonlinear Model

Given nonlinear model

\[ Z(x, u) \dot{x} = f(x, u) \]
\[ y = g(x) \]

Setting an operation point

\[ \dot{x} = Z(x, u)^{-1} f(x, u) \]
\[ = F(x, u). \]

\[ \frac{\partial F}{\partial x} \bigg|_{x_0, u_0} = Z(x_0)^{-1} \frac{\partial f}{\partial x} \bigg|_{x_0, u_0} + \frac{\partial Z^{-1}}{\partial x} \bigg|_{x_0} f(x_0, u_0) \]

\[ f(x_0, u_0) = 0 \]

\[ \delta x = \left[ \frac{\partial F}{\partial x} \bigg|_{x_0, u_0} \right] \delta x + \left[ \frac{\partial F}{\partial u} \bigg|_{x_0, u_0} \right] \delta u \]

\[ x = x_0 + \delta x \]
\[ u = u_0 + \delta u \]
Linearization of Nonlinear model

Given nonlinear model

\[ Z(x, u) \dot{x} = f(x, u) \]
\[ y = g(x) \]

\[ Z(x, u) \dot{x} = f(x, u) \]
\[ \dot{x} = Z(x, u)^{-1} f(x, u) \]
\[ = F(x, u). \]

Setting an operation point

\[ x = x_o + \delta x \]
\[ u = u_o + \delta u \]

Expanding

\[ \frac{\partial F}{\partial x} \bigg|_{x_o, u_o} = Z(x_o)^{-1} \frac{\partial f}{\partial x} \bigg|_{x_o, u_o} + \frac{\partial Z^{-1}}{\partial x} \bigg|_{x_o} f(x_o, u_o) \]
\[ f(x_o, u_o) = 0 \]

\[ A = Z(x_o)^{-1} \frac{\partial f}{\partial x} \bigg|_{x_o, u_o}, \quad B = Z(x_o)^{-1} \frac{\partial f}{\partial u} \bigg|_{x_o, u_o}, \]
\[ C = Z(x_o)^{-1} \frac{\partial g}{\partial x} \bigg|_{x_o, u_o}, \quad D = Z(x_o)^{-1} \frac{\partial g}{\partial u} \bigg|_{x_o, u_o}. \]

\[ \dot{x} = A \delta x + B \delta u \]
\[ y = C \delta x + D \delta u \]
Model order reduction

For the linearized model
\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]
reduced order model can be obtained as
\[
\begin{bmatrix}
A_r & B_r \\
C_r & D_r
\end{bmatrix} = \begin{bmatrix}
\Sigma_\Gamma^{-\frac{1}{2}}(A_{11} - A_{12}A_{22}^\dagger A_{21}) \Sigma_\Gamma^{-\frac{1}{2}} & \Sigma_\Gamma^{-\frac{1}{2}}(B_1 - A_{12}A_{22}^\dagger B_2) \\
-(C_1 - C_2A_{22}^\dagger A_{21}) \Sigma_\Gamma^{-\frac{1}{2}} & D_1 - C_2 - A_{22}^\dagger B_2
\end{bmatrix}
\]

Reduction process

1. Get controllability & observability Gramians using Lyapunov equation
   \[
   0 = AP + PA^T + BB^T, \quad 0 = QA + A^TQ + C^TC.
   \]
2. Choose the desired reduced order \( n \) and form the descriptor
   \[
   \Gamma = QP - \zeta^2I \quad \text{where} \quad \sigma_n > \zeta \geq \sigma_{n+1}
   \]
3. Taking singular value decomposition of \( \Gamma \)
   \[
   \Gamma = [U_{\Gamma 1} \quad U_{\Gamma 2}][\Sigma \quad 0 \quad 0][V_{\Gamma 1}^T \quad V_{\Gamma 2}^T]
   \]
   ※ \( A_{22}^\dagger \) is pseudo inverse of \( A_{22} \)
4. Calculate
   \[
   \begin{bmatrix}
   A_{11} & A_{12} \\
   A_{21} & A_{22}
   \end{bmatrix} = \begin{bmatrix}
   U_{\Gamma 1}^T & U_{\Gamma 2}^T \end{bmatrix} \begin{bmatrix}
   \zeta^2 A^T + QAP & [V_{\Gamma 1} \quad V_{\Gamma 2}]
   \end{bmatrix}
   \]
   \[
   \begin{bmatrix}
   B_1 \\
   B_2
   \end{bmatrix} = \begin{bmatrix}
   U_{\Gamma 1}^T \\
   U_{\Gamma 2}^T
   \end{bmatrix} \begin{bmatrix}
   QB - C^T
   \end{bmatrix}
   \]
   \[
   [C_1 \quad C_2] = \begin{bmatrix}
   CP \\
   -\zeta B^T
   \end{bmatrix} \begin{bmatrix}
   V_{\Gamma 1} \\
   V_{\Gamma 2}
   \end{bmatrix}
   \]
   \[
   D_1 = D
   \]
Nonlinear model vs. reduced linear model

< Response of pressure difference and superheat temperature >
Solid line: 4th order linear model
Dotted line: nonlinear model
Fault estimation using PI-observer

Dynamic model including fault and disturbance

\[
\dot{x}(t) = A\delta x(t) + B\delta u(t) + B_f\hat{f}(t) + Gd(t),
\]

\[
y(t) = C\delta x(t),
\]

\(B_f: \) actuator fault matrix spanned by matrix \(B\),
\(G: \) matrix which describe the external disturbance.

Proportional-Integral observer for state and fault detection

\[
\dot{\hat{x}}(t) = A\delta \hat{x}(t) + B\delta u(t) + L_P (y(t) - \hat{y}(t)) + B_f\hat{f}(t),
\]

\[
\dot{\hat{y}}(t) = C\delta \hat{x}(t),
\]

\[
\hat{f}(t) = L_I (y(t) - \hat{y}),
\]

\(\hat{x}: \) Estimate of states,
\(\hat{y}: \) Estimate of fault,
\(L_P: \) Proportional observer gain,
\(L_I: \) Integral observer gain.
Fault estimation using PI-observer

Dynamic model including fault and disturbance

\[ \dot{x}(t) = A\delta x(t) + B\delta u(t) + B_f f(t) + Gd(t), \]
\[ y(t) = C\delta x(t), \]

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\( G : \) matrix which describe the external disturbance.

Proportional-Integral observer for state and fault detection

\[ \dot{\hat{x}}(t) = A\delta \hat{x}(t) + B\delta u(t) + L_P (y(t) - \hat{y}(t)) + B_f \hat{f}(t), \]
\[ \dot{\hat{y}}(t) = C\delta \hat{x}(t), \]
\[ \hat{f}(t) = L_I (y(t) - \hat{y}), \]

\( \hat{x} : \) Estimate of states,
\( \hat{y} : \) Estimate of fault,
\( L_P : \) Proportional observer gain,
\( L_I : \) Integral observer gain.

Define state & fault estimation error

\[ e_x(t) = x(t) - \hat{x}(t), \quad e_f(t) = f(t) - \hat{f}(t) \]

Assume \( \dot{f}(t) = 0 \), then augmented error dynamics is

\[
\begin{bmatrix}
\dot{e}_x \\
\dot{e}_f
\end{bmatrix}
= \begin{bmatrix}
A - L_P C & B_f \\
-L_I C & 0
\end{bmatrix}
\begin{bmatrix}
e_x \\
e_f
\end{bmatrix}
+ \begin{bmatrix}
G \\
0
\end{bmatrix} d
\]
Fault estimation using PI-observer

Define $\bar{e}(t) = [ e_x \ e_f ]$ then

$$\dot{\bar{e}}(t) = ( \bar{A} - \bar{L}\bar{C} ) \bar{e}(t) + \bar{G}d(t)$$

where

$$\bar{A} = \begin{bmatrix} A & B_f \\ 0 & 0 \end{bmatrix}, \bar{L} = \begin{bmatrix} L_P \\ L_I \end{bmatrix}, \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \bar{G} = \begin{bmatrix} G \\ 0 \end{bmatrix}.$$ 

Using $H\infty$ robust theory, robust PI-observer can be designed as

**Theorem 1:** Given a scalar $\gamma_o > 0$, there exist an $H\infty$ observer if and only if there exists $P = P^T > 0$, $Y$ such that the following matrix inequality

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} - \bar{C}^T Y^T - Y \bar{C} & PG & I \\ \bar{C}^T P & -\gamma_o I & 0 \\ I & 0 & -\gamma_o I \end{bmatrix} < 0.$$ 

The $H\infty$ observer gain matrix is given by $\bar{L} = P^{-1}Y$. 
Observer based fault-tolerant control

Fault-tolerant controller design

System Dynamics

\[
\begin{align*}
\dot{x}(t) &= A\delta x(t) + B\delta u(t) + B_f f(t) + C d(t), \\
y(t) &= C\delta x(t), \\
\delta u(t) &= K_P \delta \dot{x}(t) + K_f \dot{f}(t)
\end{align*}
\]

Observer Dynamics

\[
\begin{align*}
\dot{x}(t) &= A\delta \dot{x}(t) + B\delta u(t) + L_P (y(t) - \hat{y}(t)) + B_f \dot{f}(t), \\
\dot{y}(t) &= C\delta \dot{x}(t), \\
\dot{f}(t) &= L_I (y(t) - \hat{y}),
\end{align*}
\]
Observer based fault-tolerant control

Fault-tolerant controller design

**System Dynamics**

\[
\begin{align*}
\dot{x}(t) &= A\delta x(t) + B\delta u(t) + B_f f(t) + C d(t), \\
y(t) &= C\delta x(t), \\
\delta u(t) &= K_P \delta \dot{x}(t) + K_f \dot{f}(t)
\end{align*}
\]

**Observer Dynamics**

\[
\begin{align*}
\dot{\hat{x}}(t) &= A\delta \hat{x}(t) + B\delta u(t) + L_P (y(t) - \hat{y}(t)) + B_f \hat{f}(t), \\
\hat{y}(t) &= C\delta \hat{x}(t), \\
\hat{f}(t) &= L_I (y(t) - \hat{y}),
\end{align*}
\]

Augmented system dynamics

\[
\begin{bmatrix}
\dot{x} \\
\dot{\varepsilon}
\end{bmatrix} =
\begin{bmatrix}
(A + BK_P) & \tilde{B} \\
0 & A - L\tilde{C}
\end{bmatrix}
\begin{bmatrix}
\delta x \\
\varepsilon
\end{bmatrix} +
\begin{bmatrix}
G' \\
\bar{G}
\end{bmatrix} d(t)
\]

where \( \tilde{B} = [0 \ B_f] \).
Observer based fault-tolerant control

Fault-tolerant controller design

System Dynamics
\[
\begin{align*}
\dot{x}(t) &= A\delta x(t) + B\delta u(t) + B_f f(t) + Gd(t), \\
y(t) &= C\delta x(t), \\
\delta u(t) &= K_P\delta x(t) + K_f \hat{f}(t)
\end{align*}
\]

Observer Dynamics
\[
\begin{align*}
\dot{x}(t) &= A\delta \hat{x}(t) + B\delta u(t) + L_P (y(t) - \hat{y}(t)) + B_f \hat{f}(t), \\
\hat{y}(t) &= C\delta \hat{x}(t), \\
\hat{f}(t) &= L_I (y(t) - \hat{y}),
\end{align*}
\]

Augmented system dynamics
\[
\begin{bmatrix}
\dot{x} \\
\dot{\bar{e}}
\end{bmatrix} =
\begin{bmatrix}
(A + BK_P) & \bar{B} \\
0 & A - \bar{L}\bar{C}
\end{bmatrix}
\begin{bmatrix}
\delta x \\
\bar{e}
\end{bmatrix} +
\begin{bmatrix}
G \\
\bar{G}
\end{bmatrix} d(t)
\]

where \( \bar{B} = [0 \quad B_f] \).

→ This means that separation property holds, so that the state-feedback gain \( K_P \) and the observer gain \( \bar{L} \) can be designed separately.
Observer based fault-tolerant control

From system dynamics and input we can get

\[ \dot{x} = (A + BK_P)\delta x + B_f e_f + Gd \]

Theorem 2: The closed loop VCC system is asymptotically stable and \( \|T_{dy}\|_\infty < \gamma_c \) with input constraint \( |\delta u_j(t)| \leq \delta u_{j,\text{max}} \) if there exist matrices \( X > 0, \bar{K} \), and \( Z = Z^T \) such that

\[
\begin{bmatrix}
\Gamma & G & X \\
G^T & -\gamma_c I & 0 \\
X & 0 & -\gamma_c I \\
\end{bmatrix} < 0, \quad \begin{bmatrix} Z & \bar{K} \\ \bar{K}^T & X \end{bmatrix} > 0,
\]

where \( \Gamma = XA^T + AX + B\bar{K} + \bar{K}^TB^T \), \( Z_{jj} \leq \delta u_{j,\text{max}}^2 \) and feedback gain \( K_P = \bar{K}X^{-1} \).
Simulation Result

< Compressor fault signal >

< System outputs >
Conclusion

• Reduced linear model of vapor compression cycle system is obtained.

• Robust Fault estimation using PI-observer designed.

• Observer-based Robust Fault-tolerant control with input constraint was proposed.

• Some method can be applied for the system to be stable globally.