Synchronization of two different chaotic systems using Terminal Sliding mode control with disturbance observer

Ban Jaepil, Lee Jinwoo, Won sangchul
POSTECH

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2. Synchronization using Terminal Sliding mode control (TSMC)
3. Synchronization using TSMC and Nonlinear disturbance observer
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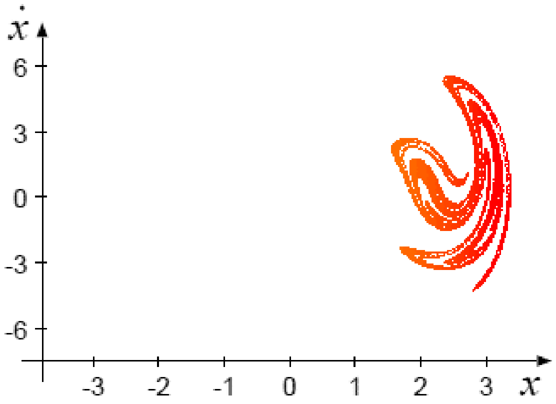
5. Conclusion

Chaotic system

- What is Chaotic system?
  Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions. (in Wikipedia)

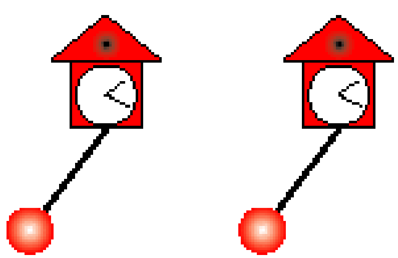


Chaotic behavior of Duffing oscillator

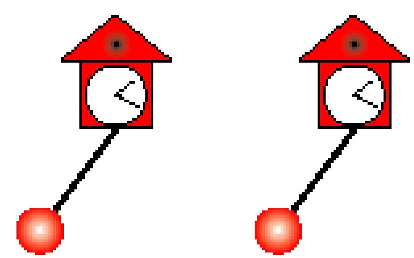
Chaotic behavior of Double rod pendulum

Synchronization of Chaotic systems

- **Synchronization** is the coordination of events to operate a system in unison. (from Greek *συν* : *syn* = the same, common and *χρονος* : *chronos*= time)



synchrony

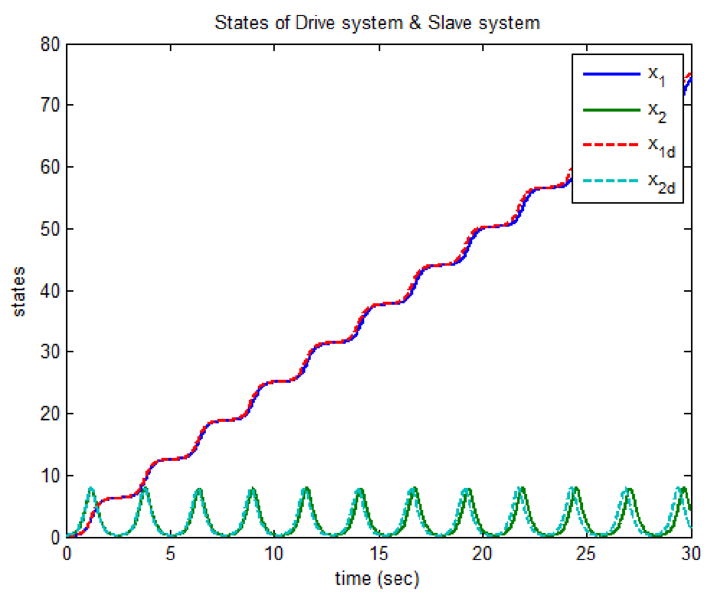


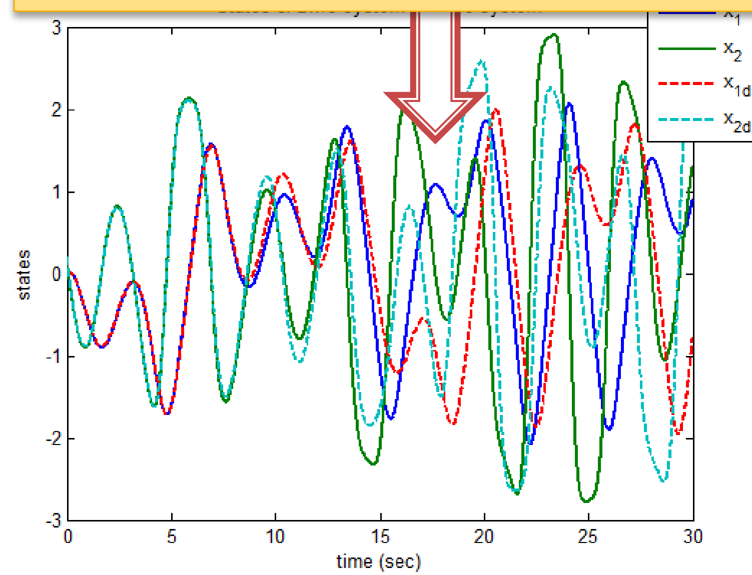
No synchrony

Synchronization of Chaotic systems

- Synchronization of chaotic syste

Behave differently
as they are different systems each other





Cart inverted pendulum

Duffing oscillator

Initial conditions : x_{10}=0.01, x_{20}=0.2

x_{1do}=0.011 , x_{2do}=0.21

Synchronization using Sliding mode control

- Consider 2nd order chaotic systems

$$\begin{cases} \dot{x}_{d1} = x_{d2} \\ \dot{x}_{d2} = f_d(x_d) \end{cases}$$ Drive system

$$\begin{cases} \dot{x}_{r1} = x_{r2} \\ \dot{x}_{r2} = f_r(x_r) + g_r(x_r)u + d \end{cases}$$ Response system

where $x_d = [x_{d1}, x_{d2}]^T, x_r = [x_{r1}, x_{r2}]^T, f_d, f_r, g_r$ are continuous function, d is a disturbance.

Synchronization using Sliding mode control

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If we define error
$e = [x_r - x_d]^T = [x_{r1} - x_{d1}, x_{r2} - x_{d2}]^T = [e_1, e_2]^T$

Synchronization using Sliding mode control

- Consider 2nd order chaotic systems

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If we define error
$e = [x_r - x_d]^T = [x_{r1} - x_{d1}, x_{r2} - x_{d2}]^T = [e_1, e_2]^T$

then we can get the error dynamics of two chaotic system

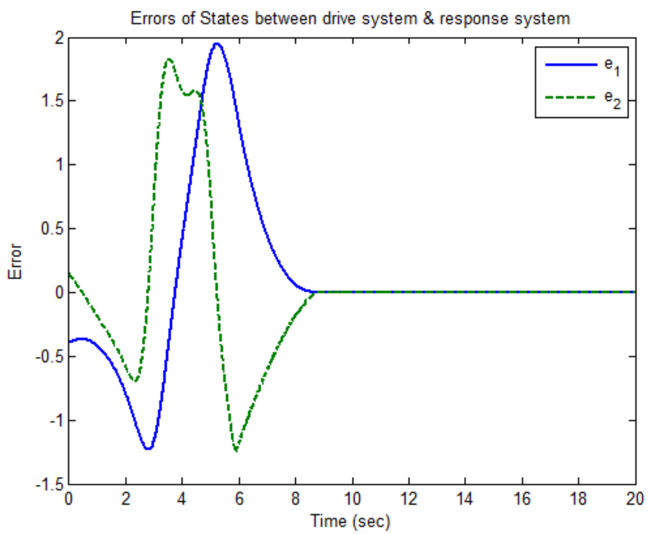
$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = f_r(x_r) - f_d(x_d) + g_r(x_r)u + d \end{cases}$$

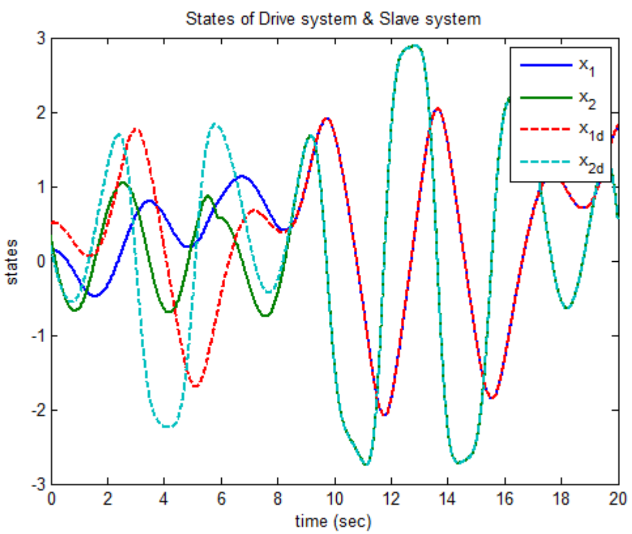
Synchronization using Sliding mode control

- Synchronization of two chaotic system

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = f_r(x_r) - f_d(x_d) + g_r(x_r)u + d \end{cases}$$

If error states goes to zero then two chaotic systems are synchronized



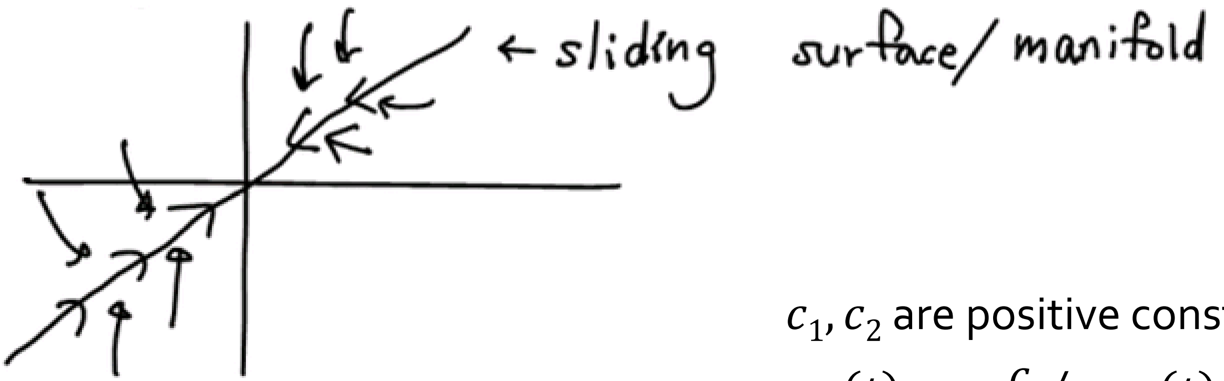


Synchronization using Sliding mode control

- Sliding mode control

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = f_r(x_r) - f_d(x_d) + g_r(x_r)u \end{cases} \Rightarrow \begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = K_f\, sgn(s) \end{cases}$$

$$u = -{g_r}^{-1}\{f_r(x_r) - f_d(x_d) + K_f sgn(s)\}$$



c_1, c_2 are positive constant

$$s(t) = c_1 e_1(t) + c_2 e_2(t) = 0$$: sliding surface

$$\Rightarrow \begin{cases} e_2(t) = -{}^{c_1}/_{c_2}\, e_1(t) \\ \dot{e}_1(t) = -{}^{c_1}/_{c_2}\, e_1(t) \end{cases}$$

1. we know how to send the states on sliding surface
2. once the states are on sliding surface and maintained , the states go to zero

Synchronization using Terminal Sliding mode control

SLIDING MODE CONTROL

- Sliding surface :

$$s(t) = c_1 e_1(t) + c_2 e_2(t)$$

- Control input :

$$u = -g_r^{-1}\{f_r(x_r) - f_d(x_d) + K_f sgn(s)\}$$

- Convergence of error states are guaranteed within infinite time

TERMINAL SLIDING MODE CONTROL

- Sliding surface :

$$s(t) = c_1 e_1^{p/q}(t) + c_2 e_2(t)$$

where $p < q$ are positive odd number

- Control input:

$$u = -g_r^{-1}\{f_r(x_r) - f_d(x_d) + p/q\, e_1^{p/q-1} e_2 + K_f sgn(s)\}$$

- Convergence of error states are guaranteed within finite time

Synchronization using Terminal Sliding mode control

SLIDING MODE CONTROL

- Sliding surface :

$$s(t) = c_1 e_1(t) + c_2 e_2(t)$$

On the sliding surface
$s(t) = 0, c_1 = c_2 = 0$

$$e_1(t) + e_2(t) = 0$$

$$\dot{e}_1(t) = -{}^{c_1}/_{c_2}\, e_1(t)$$

$$\rightarrow (t_f - ti) = -\int_{e_1(ti)}^{0} \frac{1}{e_1} de_1$$

It takes infinite time that error converges to zero

TERMINAL SLIDING MODE CONTROL

- Sliding surface :

$$s(t) = c_1 {e_1}^{p/q}(t) + c_2 e_2(t)$$

where $p < q$ are positive odd number

On the sliding surface
$s(t) = 0, c_1 = c_2 = 0$

$$c_1 {e_1}^{p/q}(t) + c_2 e_2(t) = 0$$

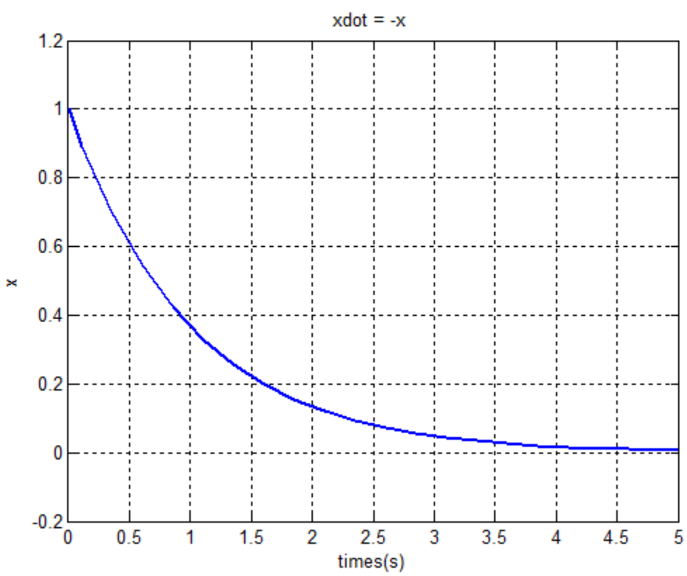
$$\dot{e}_1(t) = -{}^{c_1}/_{c_2}\, {e_1}^{p/q}(t)$$

$$\rightarrow (t_f - ti) = \frac{q}{p}\, {e_{1i}}^{p/q+1}$$

It takes finite time that error converges to zero

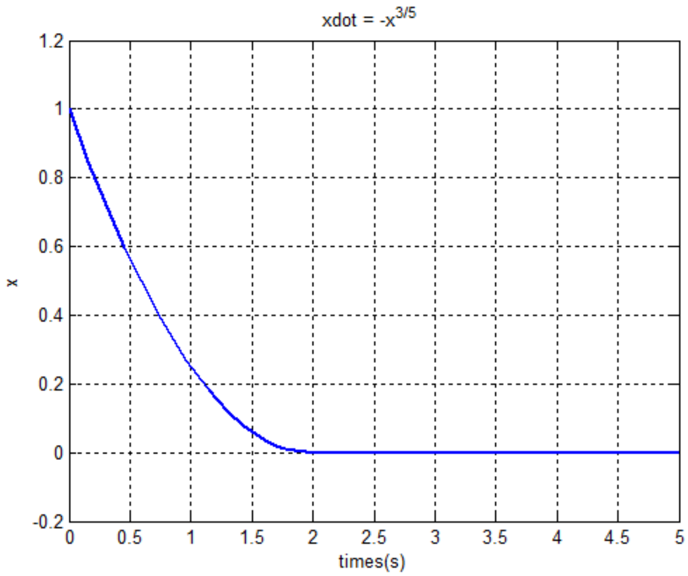
Synchronization using Terminal Sliding mode control

SLIDING MODE CONTROL



$$\dot{x} = -x$$

TERMINAL SLIDING MODE CONTROL

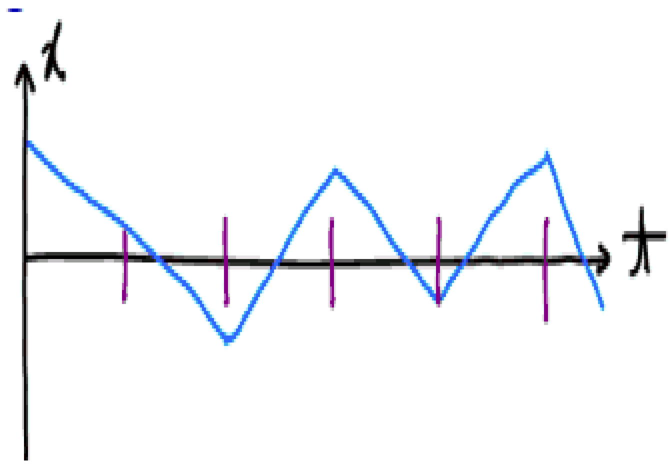


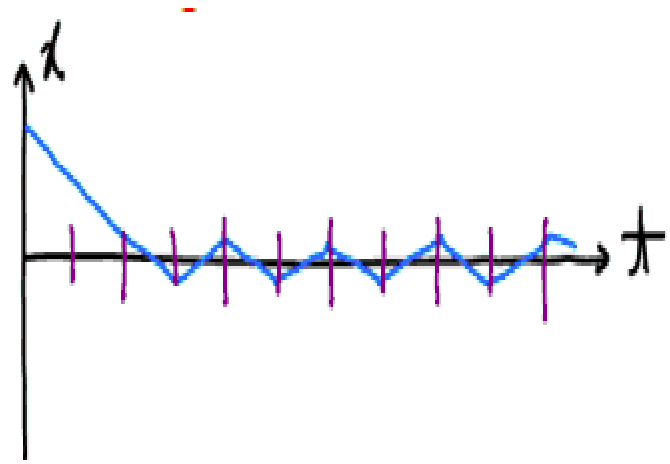
$$\dot{x} = -x^{3/5}$$

Fast convergence time

Synchronization using Terminal Sliding mode control

- Chattering of SMC





-In real digital implementation, we cannot switch control input during sampling period.

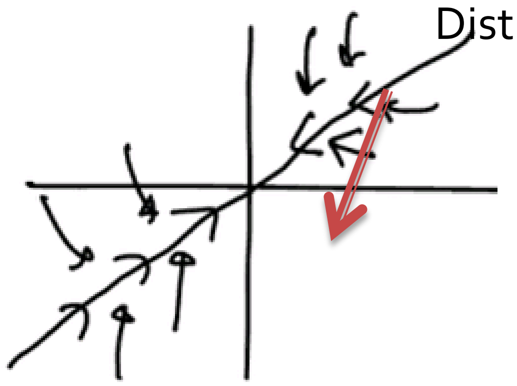
Synchronization using Terminal Sliding mode control

- 2nd order system with disturbance

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = f_r(x_r) - f_{d(x_d)} + g_{r(x_r)}u + d \end{cases} \Rightarrow \begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = K_f\, sgn(s) + d \end{cases}$$

$$u = -g_r^{-1}\{f_r(x_r) - f_d(x_d) + K_f sgn(s)\}$$

Disturbance causes the states leave from the sliding surface!



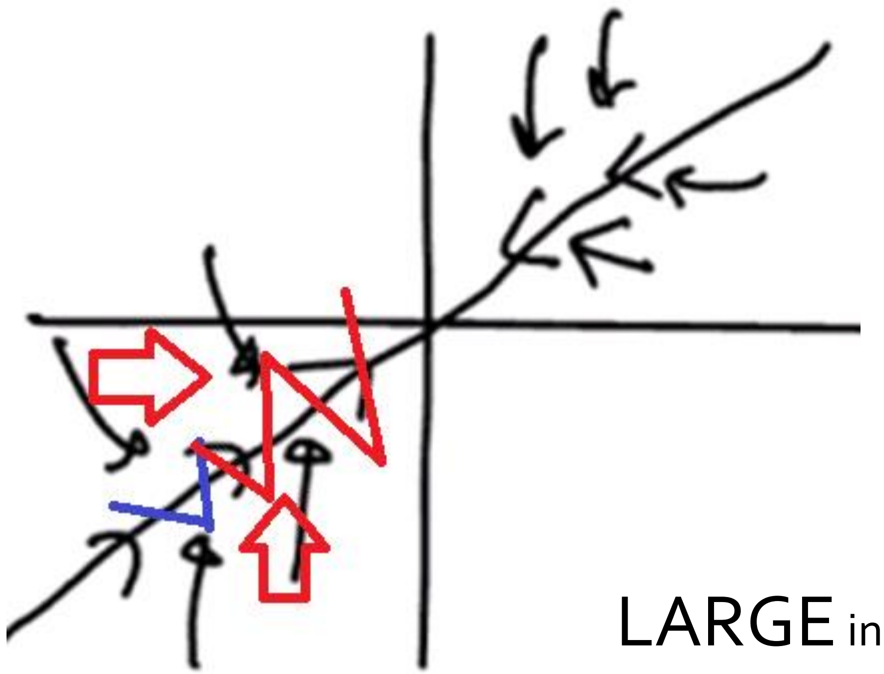
$$|d| < K_f$$

K_f has to be larger than the bound of disturbance

Ref, P. Kachroo and M. Tomizuka, "Chattering reduction and error convergence in the sliding-mode control of a class of nonlinear systems", *IEEE Transactions on Automatic Control*, Vol.41, pp.1063-1068, 1996.

Synchronization using Terminal Sliding mode control

- Chattering of SMC



LARGE input causes LARGE chattering

Nonlinear Disturbance observer

Consider the response system

$$\dot{x}_{r1} = x_{r2}$$
$$\dot{x}_{r2} = f_r(x_r) + g_r(x_r)u + d \qquad (1)$$

The nonlinear disturbance observer is

$$\dot{z} = -l(x_r)z - l(x_r)\left(p(x_r) + f_r(x_r) + g_r(x_r)u\right)$$
$$\hat{d} = z + p(x_r),$$

Wen-Hua Chen *at al* "A Nonlinear Disturbance Observer for Robotic Manipulators" IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, VOL. 47, NO. 4, AUGUST 2000

Nonlinear Disturbance observer

Solve this differential equation

$$\dot{z} = -l(x_r)z - l(x_r)\left(p(x_r) + f_r(x_r) + g_r(x_r)u\right)$$

Find estimate of *d* from this

$$\hat{d} = z + p(x_r),$$

$l(x_r)$: gain of NDOB
$p(x_r)$: function of x_{r1}, x_{r2}
z : auxiliary variable

Nonlinear Disturbance observer

Consider the response system

$$\dot{x}_{r1} = x_{r2}$$

$$\dot{x}_{r2} = f_r(x_r) + g_r(x_r)u + d \qquad (1)$$

Initial setting of estimate of *d*

$$\dot{\hat{d}} = -l(x_r)\hat{d} + l(x_r)(\dot{x}_{r2} - f_r(x_r) - g_r(x_r)u)$$

Assume that disturbance is slowly time varying

$$\dot{d} \cong 0.$$

Define

$$\tilde{d} = d - \hat{d} \qquad (2)$$

Nonlinear Disturbance observer

Define auxiliary variable

$$z = \hat{d} - p(x_r) \qquad (3)$$

Design l(x) to be

$$l(x_r) = \frac{\partial p(x_r)}{\partial x_r} \qquad (4)$$

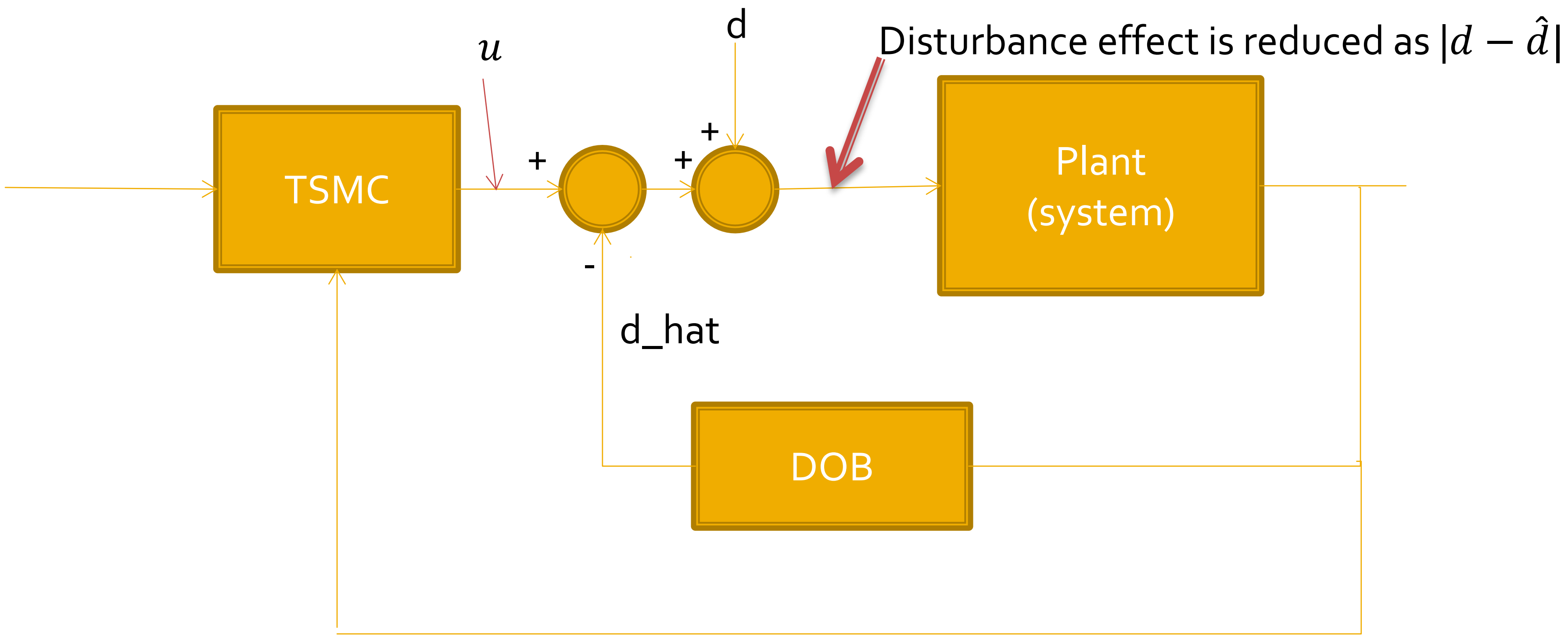
Ex1) p(x)= 10x_2
then, l(x)=10.
Ex2)p(x)=sin(x_1)
then, l(x)=cos(x_1)

…

$$\dot{z} = -l(x_r)z - l(x_r)\left(p(x_r) + f_r(x_r) + g_r(x_r)u \right)$$

$$\hat{d} = z + p(x_r),$$

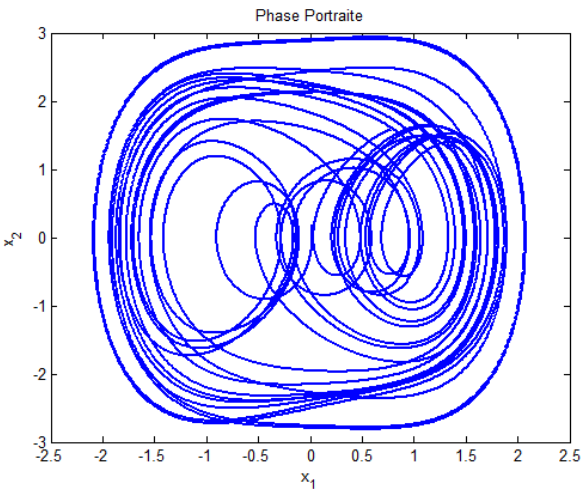
Synchronization of chaotic systems using TSMC with NDOB

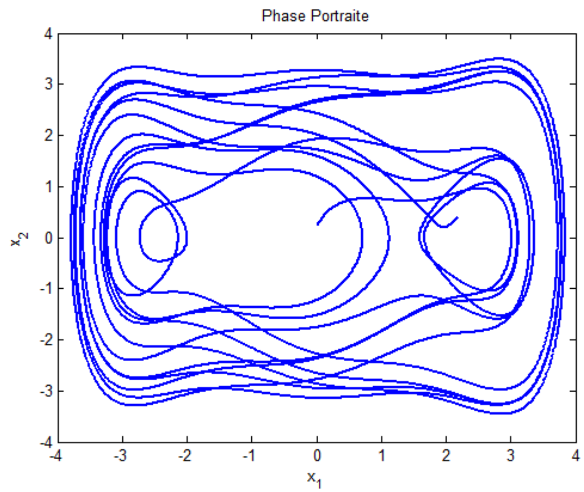


$$|d| < Kf \Rightarrow |d - \hat{d}| < \widetilde{K_f},\quad \widetilde{K_f} < Kf$$

Synchronization of chaotic systems using TSMC with NDOB

- Numerical example
  - Drive system : Duffing oscillator
  - Response system : Φ_6 Duffing oscillator





$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.4x_2 + 1.1x_1 - x_1^3 - 2.1\cos(1.8t) \end{cases}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - 0.01x_2 + 0.495x_1^3 - 0.05x_1^5 \\ \qquad + 0.78\cos(0.55t) + d + u \end{cases}$$

Synchronization of chaotic systems using TSMC with NDOB

- Error dynamics

$$\dot{e}_1 = \dot{x}_{r1} - \dot{x}_{d1} = e_2$$

$$\dot{e}_2 = \dot{x}_{r2} - \dot{x}_{d2}$$
$$= f_r(x_r) - fd(x_d) + u + d$$

$$\Rightarrow \quad \dot{e}_1 = e_2$$

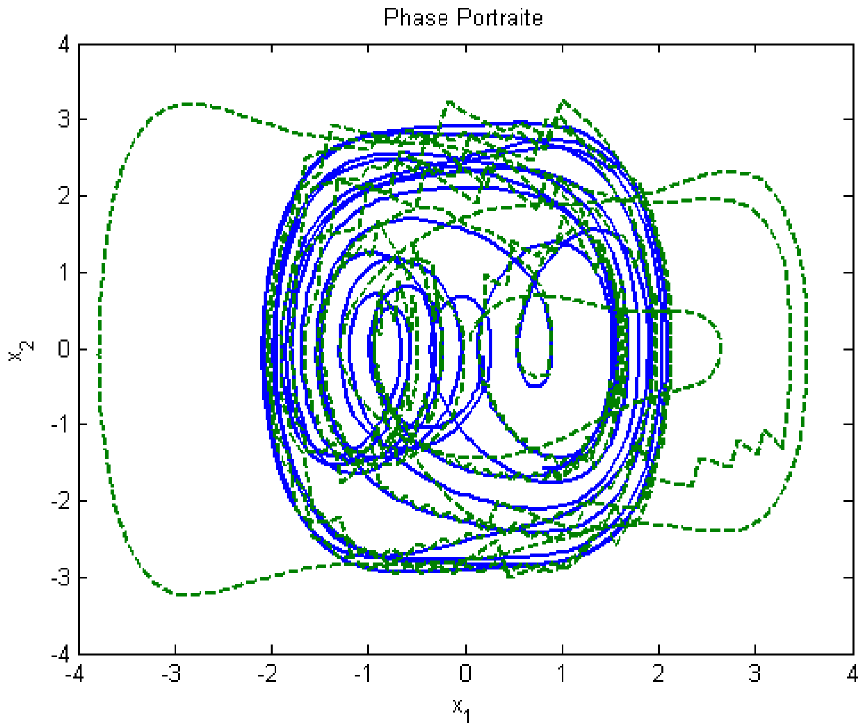
$$\dot{e}_2 = f_r(x_r) - fd(x_d) + u + d$$

If we drive e_1, e_2 goes to zero, then two chaotic systems are synchronized

$$f_d(x_d) = -0.4xd_2 + 1.1xd_1 - x_1^3 - 2.1\cos(1.8t)$$
$$f_r(x_r) = -xr_1 - 0.01xr_2 + 0.495xr_{13} - 0.05xr_{15}$$
$$+0.78\cos(0.55t)$$

Synchronization of chaotic systems using TSMC with NDOB

- Synchronization using TSMC only



$$\dot{e}_1 = e_2$$

$$\dot{e}_2 = f_r(x_r) - fd(x_d) + u + d$$

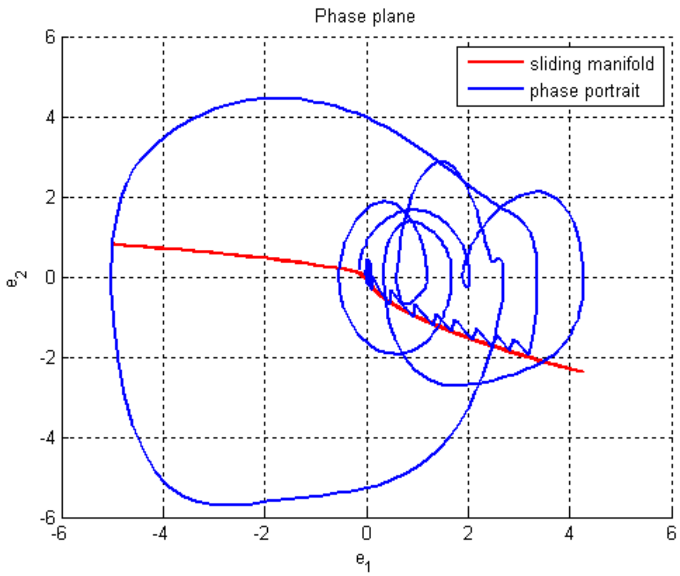
Constant disturbance applied
d=10 at t=30

Switching gain
K_f=12 (> d = 10)

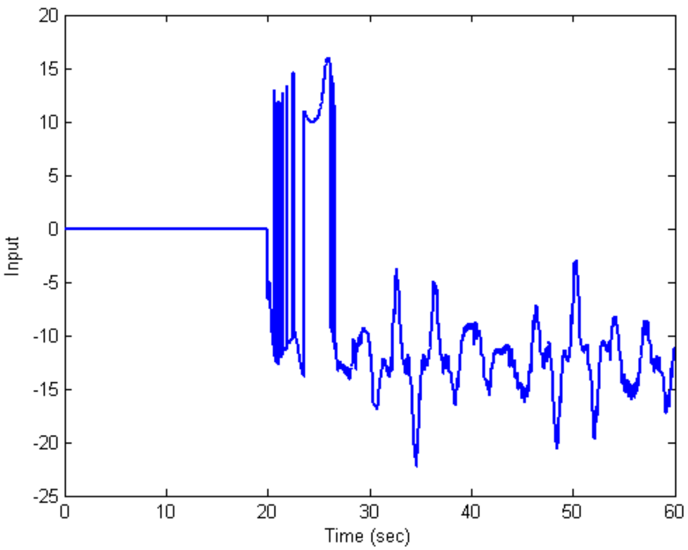
Initial conditions
$x_{d0} = [0.2, -0.1]^T$
$x_{r0} = [0.05, 0.1]^T$

Synchronization of chaotic systems using TSMC with NDOB

- Synchronization only using TSMC



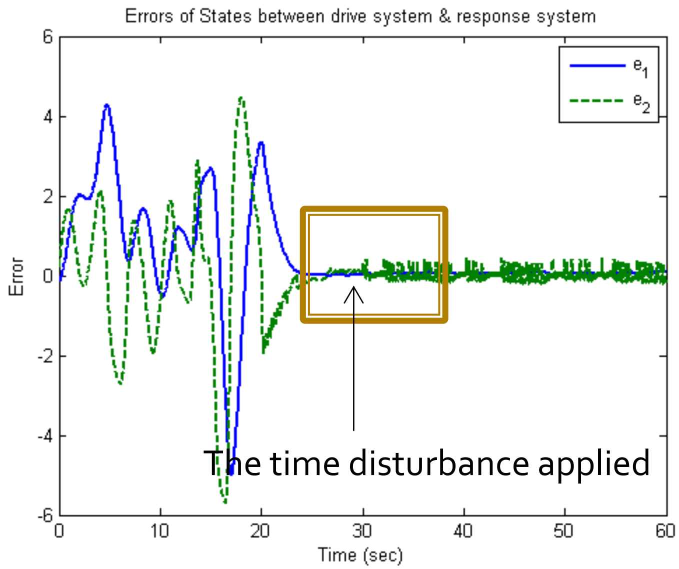
State trajectory of error dynamics

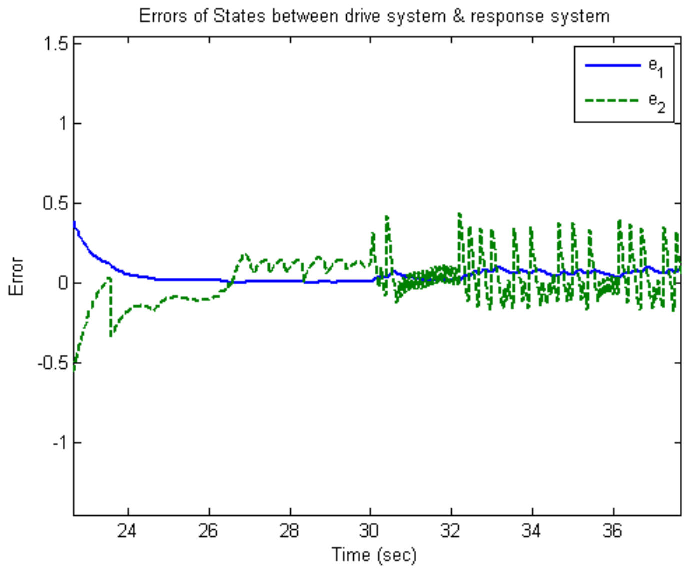


Control input

Synchronization of chaotic systems using TSMC with NDOB

- Synchronization using only TSMC

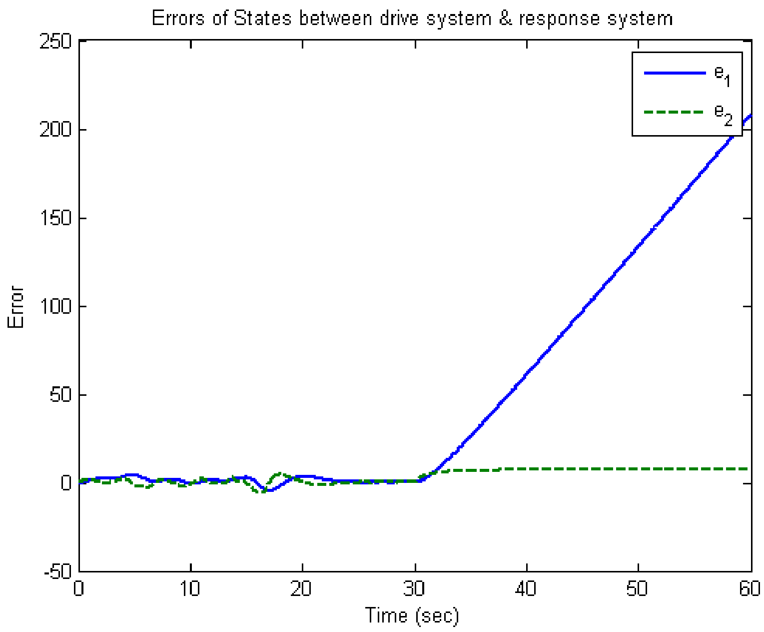




The error states vs. time

Synchronization of chaotic systems using TSMC with NDOB

- Synchronization using only TSMC



The error states vs. time

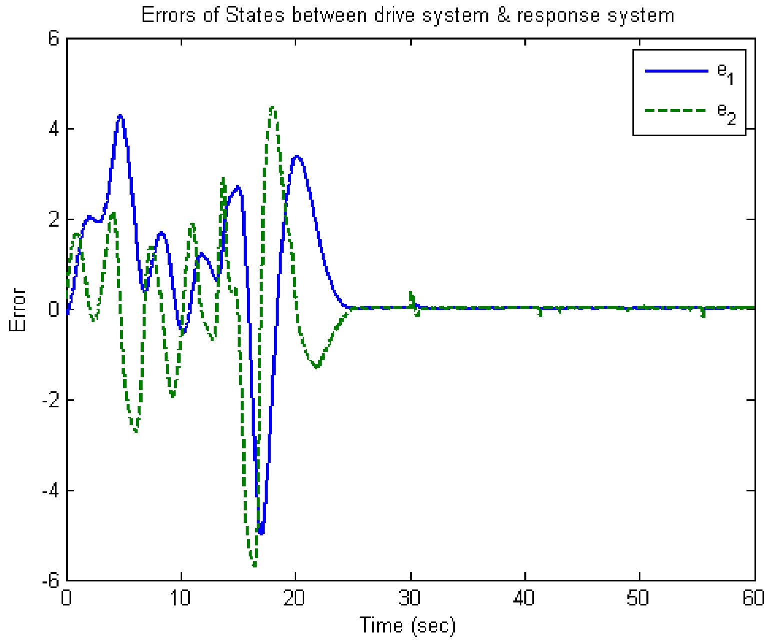
Constant disturbance applied d=10 at t=30

Switching gain K_f=2 (< d = 10)

The systems cannot be synchronized using this switching gain

Synchronization using TSMC with NDOB

- Synchronization using TSMC with DOB



The error states vs. time

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Synchronization using TSMC with NDOB

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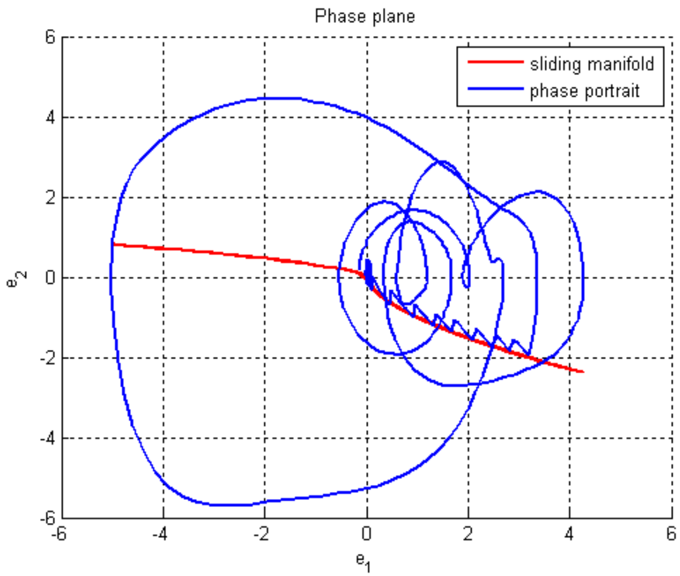
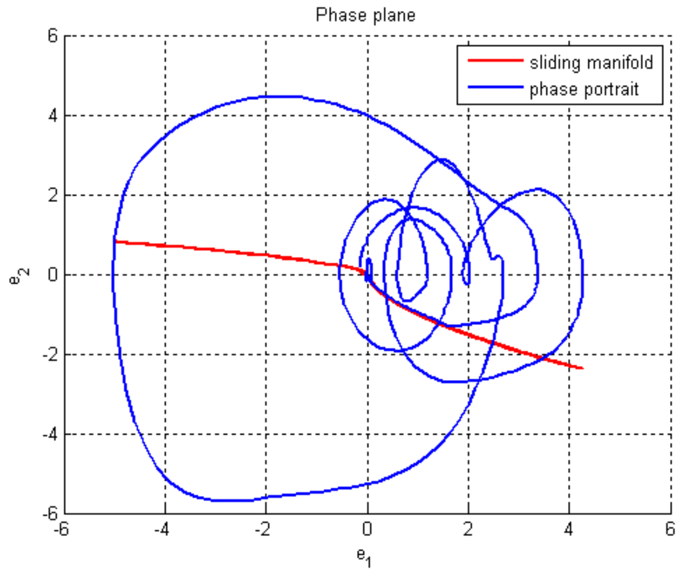


Synchronization without DOB

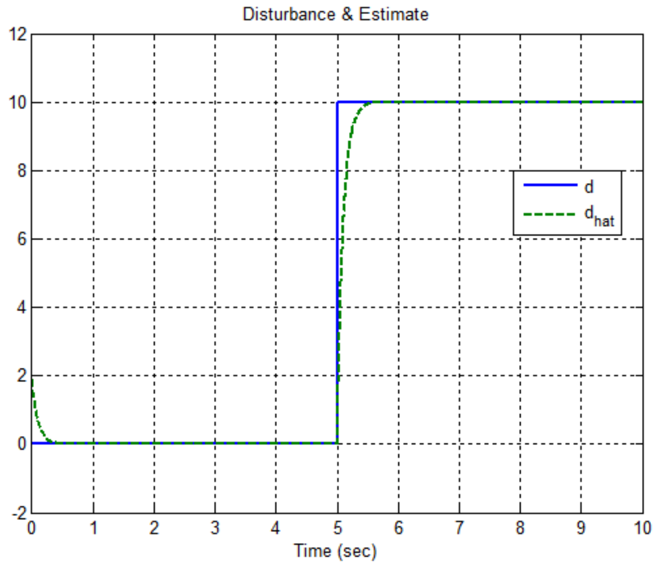


Synchronization with DOB

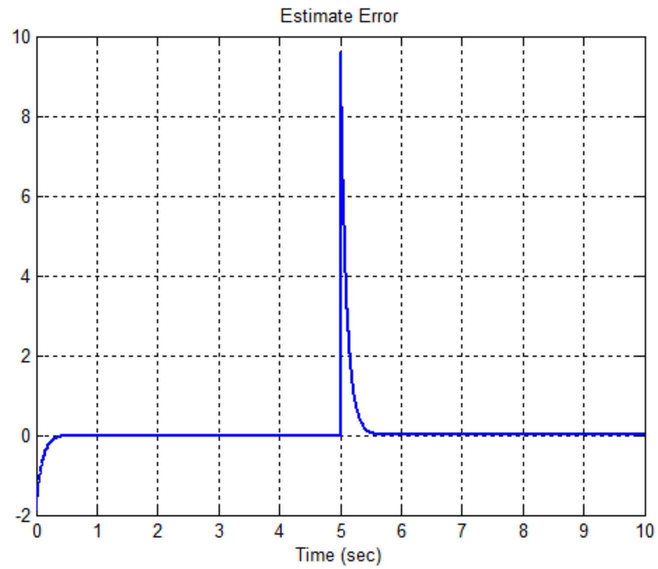
chattering is reduced

Synchronization using TSMC with NDOB

- Synchronization using TSMC with DOB



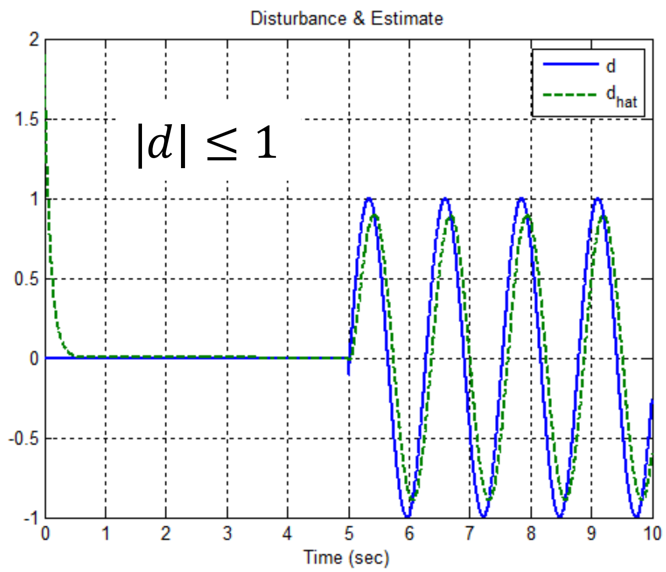
<Disturbance & estimate>



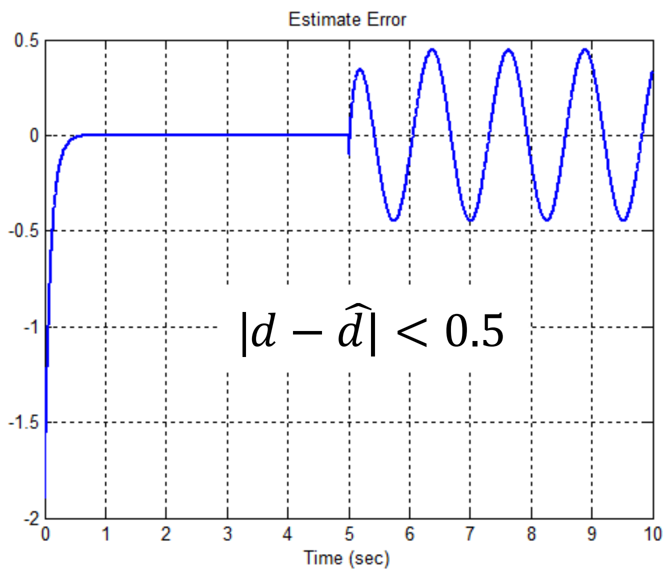
< Estimation error >

Synchronization using TSMC with NDOB

- Disturbance estimation of time-varying d



<Disturbance & estimate>



< Estimation error >

Conclusion

- Synchronization using TSMC can be failed if a large disturbance is applied.
- Disturbance observer cancels the disturbance so that the problem is solved and the chattering of TSMC reduced
- Singularity problem of terminal sliding mode control exists when synchronize the systems
- If $|d - \hat{d}|$ is unknown, K_f cannot be chosen.
  In this case, we need to develop adaptive law for K_f.