Design of Reset control by using PWQ Lyapunov function

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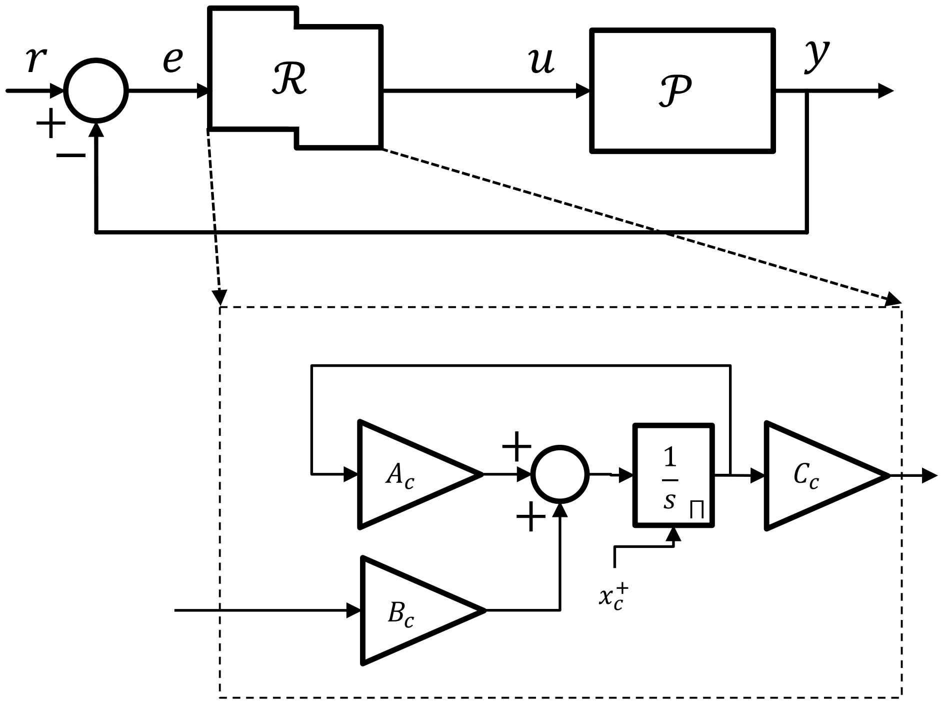
ICSL 지능제어 시스템 연구실 Intelligent Control and System Lab

Design of Reset Control?

Reset Controller

$$\dot{x}_c = A_c x_c + B_c e, \quad (u,e) \notin R$$
$$x_c^+ = A_r x_c \qquad (u,e) \in R$$
$$u_c = C_c x_c$$



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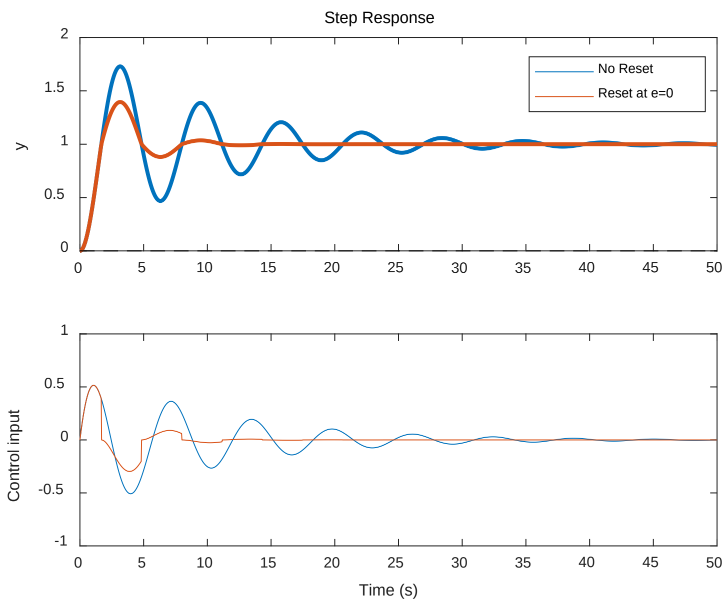

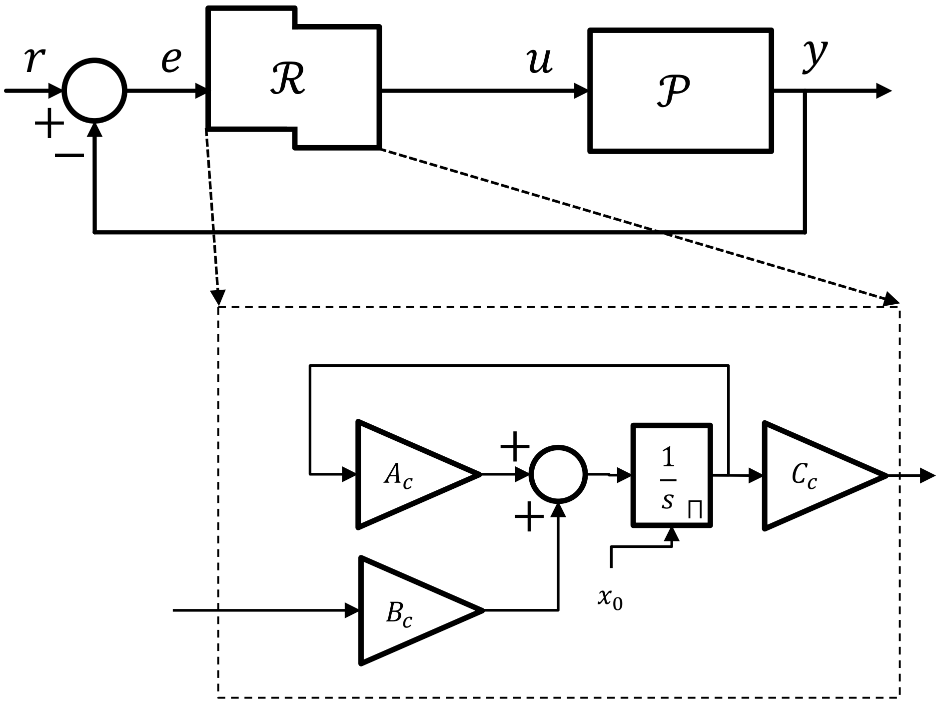

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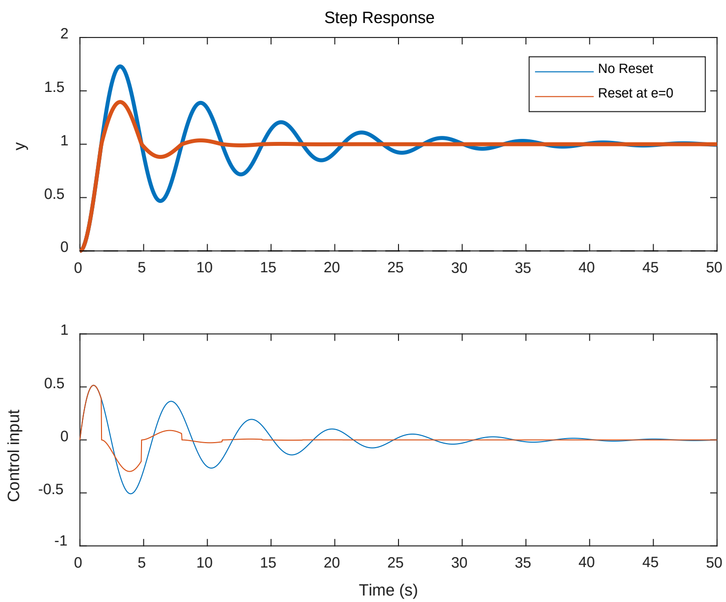

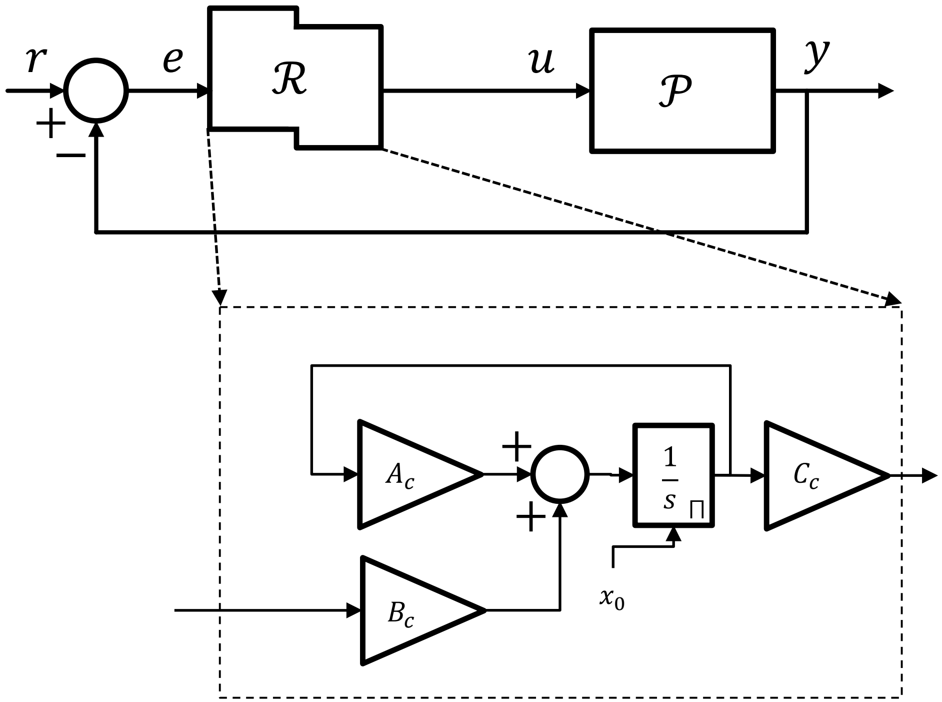

Less L2 gain (Guanglei Zhao et. al., 2016)
Phase lead (Yuqian Guo et. al., 2009)

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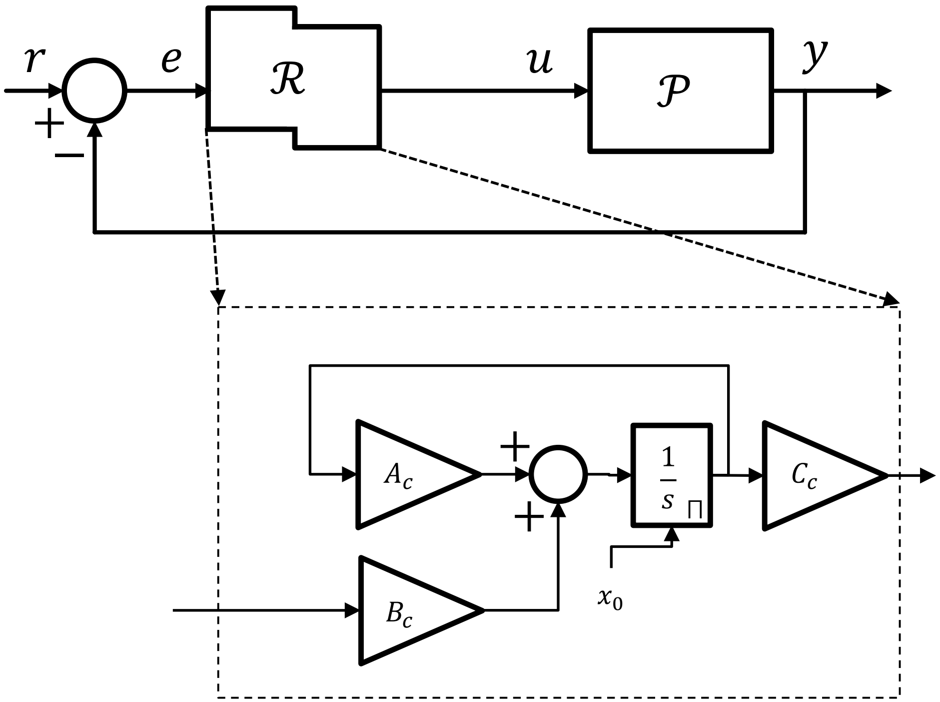

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Design of Reset Control?

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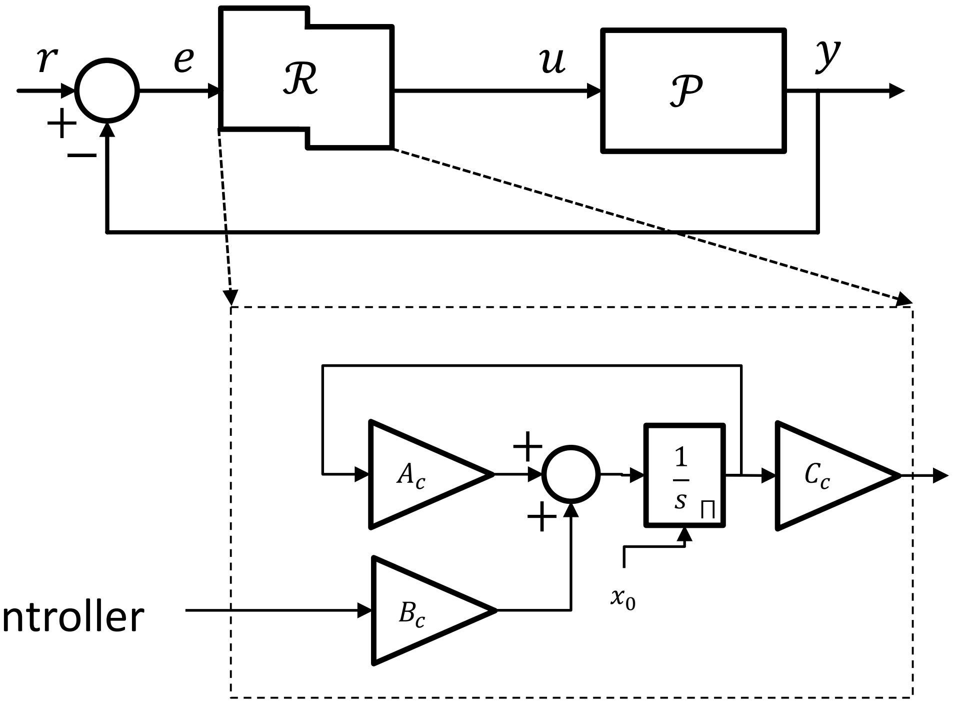

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  - ✓ Parameters of baseline linear controller

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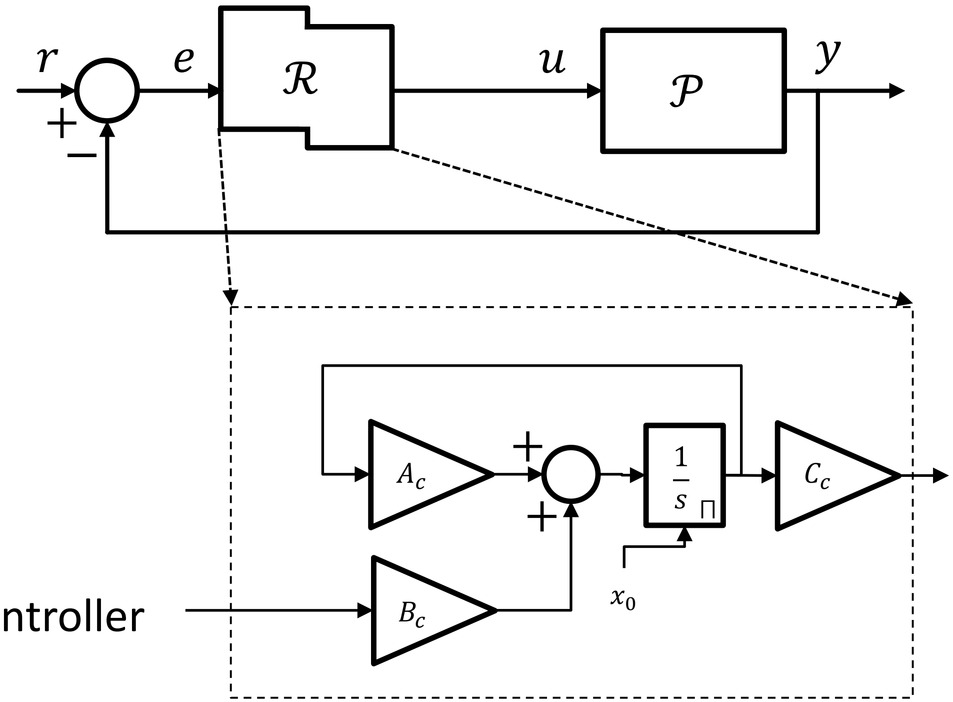

What to design?

- Baseline Linear Controller
  - ✓ Parameters of baseline linear controller
- Reset law
  - ✓ Where to reset?
  - ✓ When to reset?

Design of Reset Control?

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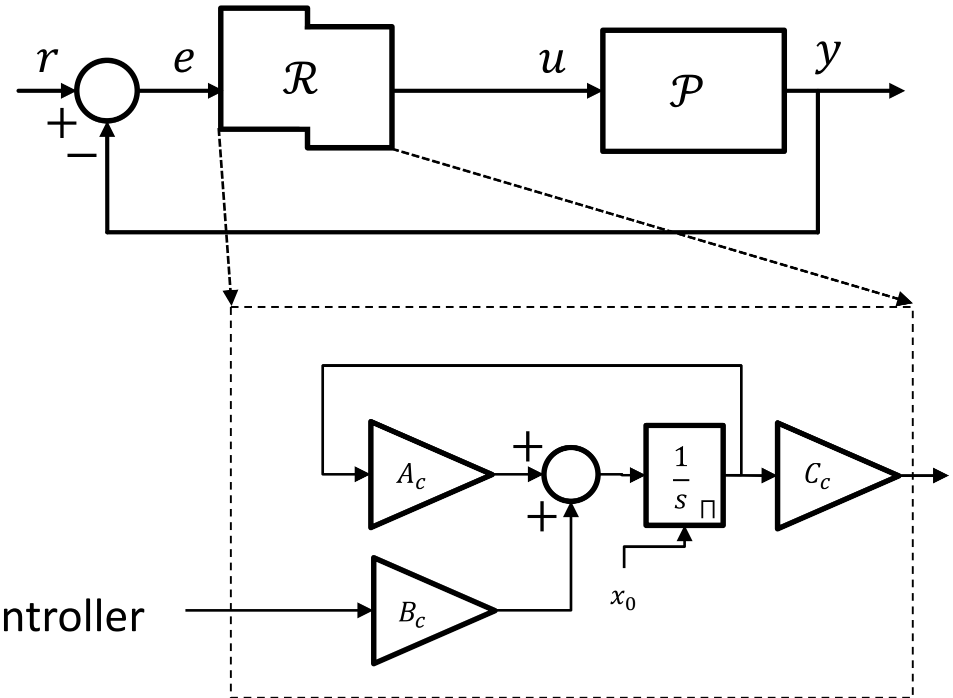

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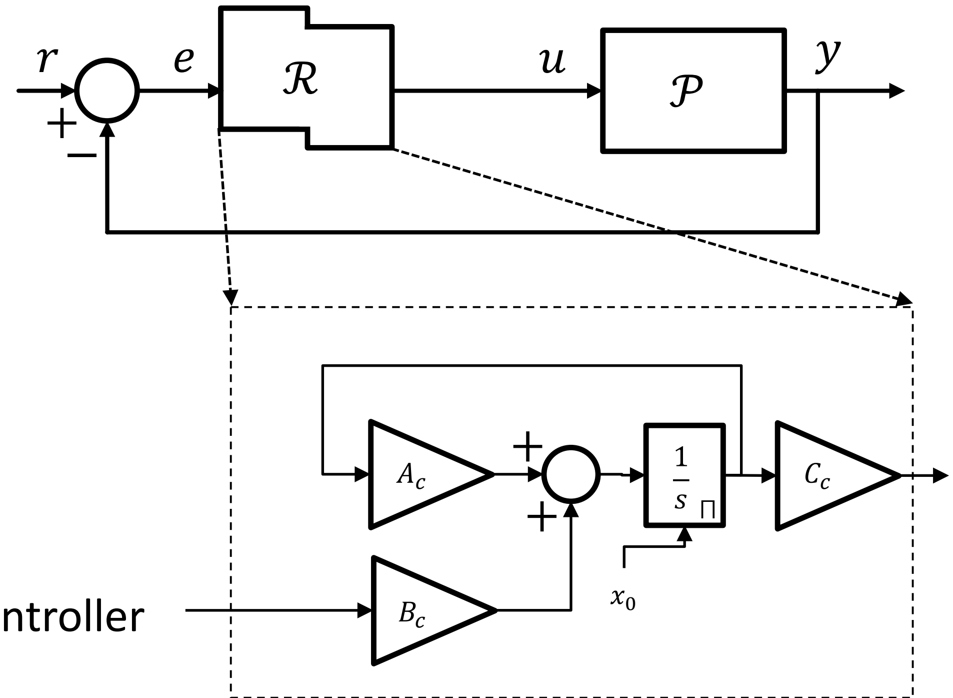

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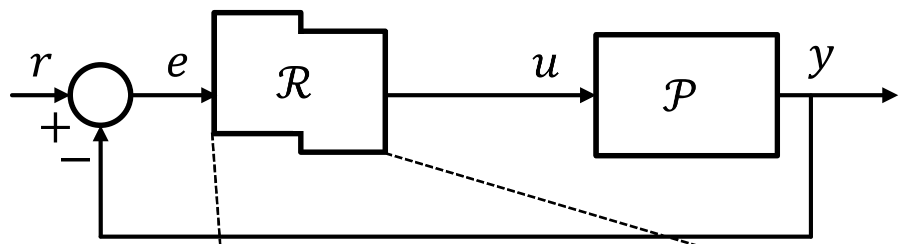

What to design?

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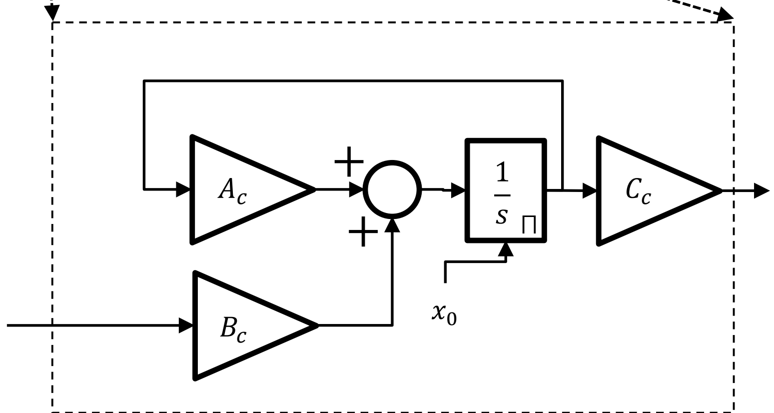

- Reset law
  - ✓ Where to reset? A_r is given (e.g., 0)
  - ✓ When to reset? When $ue \geq 0$

1) In practical application, a simple strategy is usually demanded
2) Theoretically difficult

Methods for designing a controller

Usual design approach for controller

- Design in frequency domain
  - Based on describing function
  - Bode plot (Guo, Yuqian, et al., IEEE Trans. Control Syst. Technol., 2009)
  - Nyquist plot (Van Loon, S. J. L. M., et al., Automatica, 2017)
- Design in time domain with linear matrix inequalities (LMIs)
  - Based on the stability analysis with Lyapunov stability theorem
  - Obtain controller parameters numerically in polynomial time
  - Associate with useful performance index such as H_2 and H_∞ norm

Goal: Derive LMI-based design condition of reset controller

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  - Hybrid dynamical system
  - PWQ Lyapunov function
  - Linear matrix inequalty
- LMI-based design of Reset Control
- Numerical Example
- Conclusion

Hybrid Dynamical systems

Hybrid dynamical system framework

$$\dot{x} \in F(x), \qquad x \in \mathcal{F}$$

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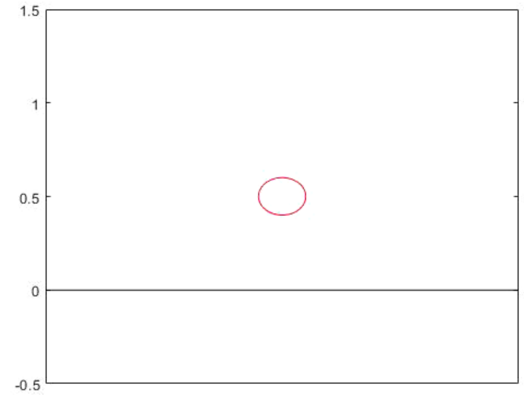
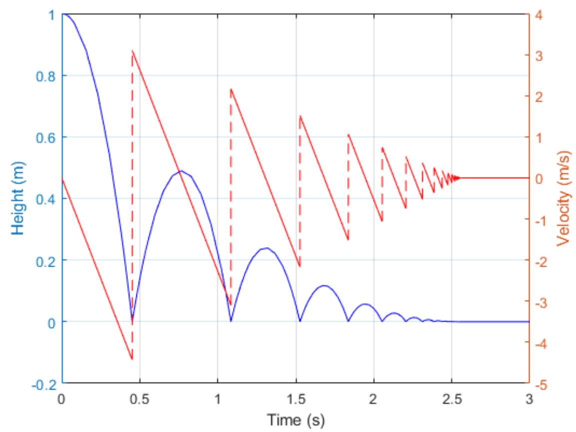


Bouncing ball example

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -g \end{aligned} \qquad \mathcal{F} := \{x | x_1 > 0\}$$

$$\begin{aligned} x_1^+ &= 0 \\ x_2^+ &= -c x_2 \end{aligned} \qquad \mathcal{J} := \{x |\, x_1 = 0\}$$

x_1: Height of the ball
x_2: Velocity of the ball

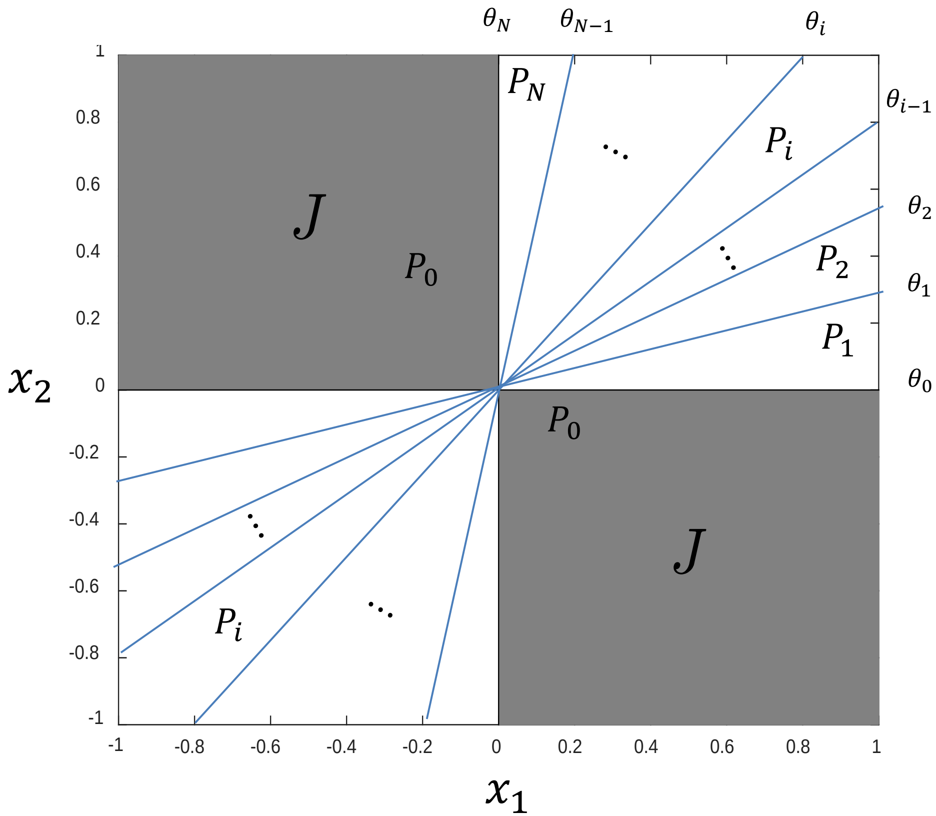


Piecewise quadratic Lyapunov function

- Piecewise quadratic Lyapunov function

$$V = x^T \mathbf{P(x)} x$$

$$x^T P_i x = x^T P_j x, \quad \forall x \in X_i \cap X_j$$



Linear matrix inequalities

- A linear matrix inequality (LMI) in the variable $x \in \mathbb{R}^n$ has the form

$$F(x) \coloneqq F_0 + x_1 F_1 + x_2 F_2 + \cdots + x_n F_n \succcurlyeq 0 (\preccurlyeq 0),$$

where $F_0 \in \mathbb{R}^{m \times m}, \ldots, F_n \in \mathbb{R}^{m \times m}$ are symmetric matrices.

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Example of LMIs (Lyapunov stability)

$$A^T \boldsymbol{P} + \boldsymbol{P} A < 0, P > 0$$

where, P is matrix variables. (A is stable if and only if there exist such P.)

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Example of bilinear matrix inequality (Static output feedback control case)

$$A^T \boldsymbol{P} + \boldsymbol{P} A + \boldsymbol{P} B \boldsymbol{K} C + C^T \boldsymbol{K}^T B^T \boldsymbol{P} < 0, P > 0$$

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LMIs can be solved by the semi-definite programming in polynomial time

Useful SDP solvers: cvx (http://cvxr.com/cvx/), MPT toolbox(https://www.mpt3.org/), MATLAB LMI toolbox

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- Motivation
- Background
- **LMI-based design of Reset Control**
  - Difficulties in deriving LMI-based design condition
  - Our Method
- Numerical Example
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Difficulties in designing reset control

- Stability condition by using PQLF (Luca Zaccarian et. al., 2005, Nesic et. Al., 2011)

$$V = x^T P(x) x$$
$$\dot{V} < 0, \quad x \in \mathcal{F}$$
$$V(x^+) - V(x) \le 0, \quad x \in \mathcal{J}$$

$$\dot{x} = Ax \qquad \mathcal{F}_i \coloneqq \{x | x^T M_i x \ge 0\}$$
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$$A = \begin{bmatrix} A_p & B_p \boldsymbol{C_c} \\ \boldsymbol{B_c} C_p & \boldsymbol{A_c} \end{bmatrix}$$
$$A_r = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$$
$$x = \begin{bmatrix} x_p^T & x_c^T \end{bmatrix}^T$$

$$M_i = \Theta_{i-1}\Theta_i^T + \Theta_i \Theta_{i-1}^T, \; i = 1, \dots, N$$
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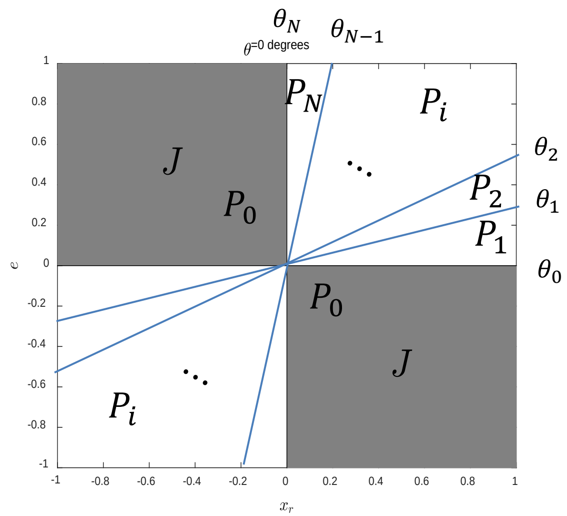

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$$\Leftrightarrow \dot V = \dot x^T P_i x + x^T P_i \dot x < 0, \qquad \forall x \in \mathcal{F}_i$$

$$V(x^+) - V(x) = (x^+)^T P_N x^+ - x^T P_0 x \le 0, \ \forall x \in \mathcal{J}$$

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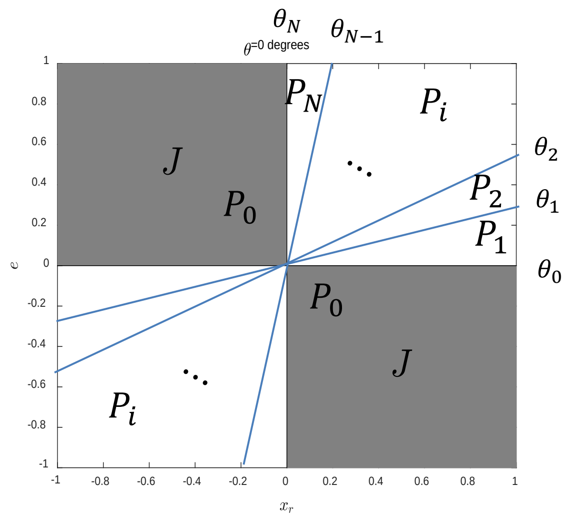

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$$\Leftrightarrow \dot{V} = \dot{x}^T P_i x + x^T P_i \dot{x} < 0, \qquad \forall x \in \mathcal{F}_i$$
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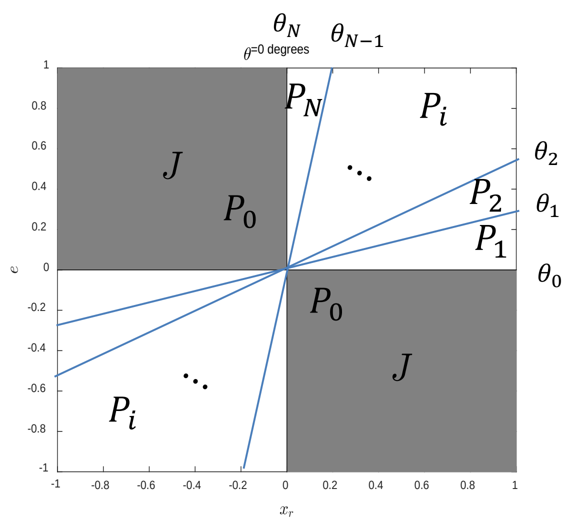
$$\Leftrightarrow x^T\left(A^T P_i + P_i A + \tau_{F_i} M_i\right)x < 0,$$
$$x^T(A_r^T P_N A_r - P_0 + \tau_R M_0)x \le 0,$$
$$\Theta_{i\perp}^T (P_i - P_{i-1})\Theta_{i\perp} = 0,$$



$\Theta_{i\perp}$: the basis of the subspace $x \in X_i \cap X_j$

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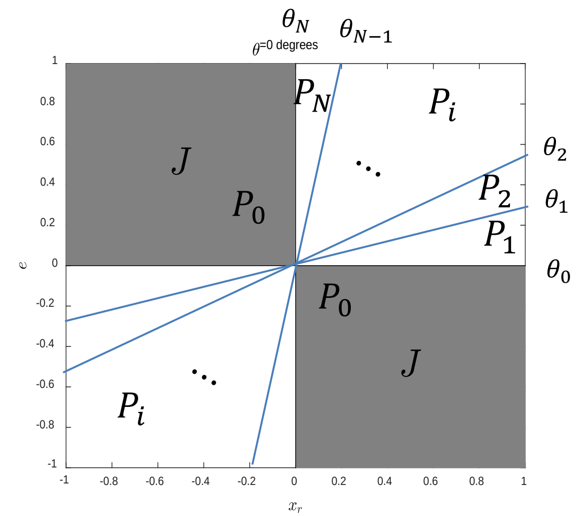
LMI-based stability condition



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Difficulties in designing reset control

- LMI-based design method for linear controller (Carsten W. Scherer et. al., 1997)

$$V = x^T P x,\ P > 0.$$

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$$\dot{V} = x^T(PA + A^T P)x < 0$$

$$\Leftrightarrow PA + A^T P < 0$$

BMIs

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$$V = x^T P x, P > 0.$$

$$\dot{V} = x^T(PA + A^T P)x < 0$$

$$\Leftrightarrow PA + A^T P < 0$$ BMIs

Define

$$P = \begin{bmatrix} S & N \\ N^T & \hat{X} \end{bmatrix}, P^{-1} = \begin{bmatrix} R & M \\ M^T & \hat{Y} \end{bmatrix}, Z_1 = \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix}, Z_2 = \begin{bmatrix} I & S \\ 0 & N^T \end{bmatrix}$$

$$\Leftrightarrow Z_1^T PAZ_1 + Z_1^T A^T PZ_1 < 0$$

$$Z_1^T PAZ_1 = \begin{bmatrix} A_pR + B_pC_cM^T & A_p \\ SA_pR + NB_cC_pR + SB_pC_cM^T + NA_cM^T & SA_p + NB_cC_p \end{bmatrix}$$

$$Z_1^T PZ_1 = \begin{bmatrix} \boldsymbol{R} & I \\ I & \boldsymbol{S} \end{bmatrix} > 0$$

Difficulties in designing reset control

- LMI-based design method for linear controller (Carsten W. Scherer et. al., 1997)

$$V = x^T P x,\ P > 0.$$

$$\dot{V} = x^T (PA + A^T P) x < 0$$

$$\Leftrightarrow PA + A^T P < 0$$ BMIs

$$\dot{x} = Ax$$

$$A = \begin{bmatrix} A_p & B_p \boldsymbol{C_c} \\ \boldsymbol{B_c} C_p & \boldsymbol{A_c} \end{bmatrix}$$

$$x = \begin{bmatrix} x_p^T & x_c^T \end{bmatrix}^T$$

$$\Leftrightarrow Z_1^T P A Z_1 + Z_1^T A^T P Z_1 < 0$$

Define

$$P = \begin{bmatrix} S & N \\ N^T & \hat{X} \end{bmatrix},\ P^{-1} = \begin{bmatrix} R & M \\ M^T & \hat{Y} \end{bmatrix},\ Z_1 = \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix},\ Z_2 = \begin{bmatrix} I & S \\ 0 & N^T \end{bmatrix}$$

$$Z_1^T P A Z_1 = \begin{bmatrix} A_p R + B_p C_c M^T & A_p \\ S A_p R + N B_c C_p R + S B_p C_c M^T + N A_c M^T & S A_p + N B_c C_p \end{bmatrix}$$

$$Z_1^T P Z_1 = \begin{bmatrix} \boldsymbol{R} & I \\ I & \boldsymbol{S} \end{bmatrix} > 0$$

Variable replacement

$$= \begin{bmatrix} A_p \boldsymbol{R} + B_p \hat{\boldsymbol{C}}_c & A_p \\ \hat{\boldsymbol{A}}_c & \boldsymbol{S} A_p + \hat{\boldsymbol{B}}_c C_p \end{bmatrix}$$

LMIs

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$$P = \begin{bmatrix} S & N \\ N^T & \hat{X} \end{bmatrix}, P^{-1} = \begin{bmatrix} R & M \\ M^T & \hat{Y} \end{bmatrix}, Z_1 = \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix}, Z_2 = \begin{bmatrix} I & S \\ 0 & N^T \end{bmatrix}$$

$$\Leftrightarrow Z_1^T PAZ_1 + Z_1^T A^T P Z_1 < 0$$

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Variable replacement

$$= \begin{bmatrix} A_p \boldsymbol{R} + B_p \widehat{\boldsymbol{C}}_c & A_p \\ \widehat{\boldsymbol{A}}_c & \boldsymbol{S} A_p + \widehat{\boldsymbol{B}}_c C_p \end{bmatrix}$$

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LMIs

Stability condition for reset control

$$A^T P_i + P_i A + \tau_{F_i} M_i < 0,$$
$$A_r^T P_N A_r - P_0 + \tau_R M_0 \leq 0,$$
$$\Theta_{i\perp}^T (P_i - P_{i-1}) \Theta_{i\perp} = 0,$$

Difficulties in designing reset control

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$$P = \begin{bmatrix} S & N \\ N^T & \hat{X} \end{bmatrix}, P^{-1} = \begin{bmatrix} R & M \\ M^T & \hat{Y} \end{bmatrix}, Z_1 = \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix}, Z_2 = \begin{bmatrix} I & S \\ 0 & N^T \end{bmatrix}$$

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Variable replacement

$$= \begin{bmatrix} A_p \boldsymbol{R} + B_p \boldsymbol{\hat{C}_c} & A_p \\ \boldsymbol{\hat{A}_c} & \boldsymbol{S} A_p + \boldsymbol{\hat{B}_c} C_p \end{bmatrix}$$

$$Z_1^T P Z_1 = \begin{bmatrix} \boldsymbol{R} & I \\ I & \boldsymbol{S} \end{bmatrix} > 0$$

LMIs

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$$A_r^T P_N A_r - P_0 + \tau_R M_0 \le 0,$$
$$\Theta_{i\perp}^T (P_i - P_{i-1}) \Theta_{i\perp} = 0,$$

M_i is indefinite matrix

Relaxing these BMIs into LMIs is extremely difficult

$$\Leftrightarrow Z_{1i}^T P_i A Z_{1i} + Z_{1i}^T A^T P_i Z_{1i} + \tau_{F_i} \boldsymbol{Z_{1i}^T M_i Z_{1i}} < 0$$
$$Z_{10}^T A_r^T P_N A_r Z_{10} - Z_{10}^T P_0 Z_{10} + \tau_R \boldsymbol{Z_{10}^T M_0 Z_{10}} \le 0,$$
$$\boldsymbol{\Theta_{i\perp}^T (P_i - P_{i-1}) \Theta_{i\perp} = 0}$$

Derive LMI condition for reset control

- Descriptor system approach

Plant dynamics

$$\dot{x}_p = A_p x_p + B_p u$$
$$y = C_p x_p$$

Controller dynamics

$$\dot{x}_c = A_c x_c + B_c y \qquad x_c^+ = 0$$
$$u = C_c x_c$$
$$0 \cdot \dot{u} = C_c x_c - u$$

$$\underbrace{\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{E} \begin{bmatrix} \dot{x}_p \\ \dot{x}_c \\ \dot{u} \end{bmatrix} = \underbrace{\begin{bmatrix} A_p & 0 & B_p \\ \boldsymbol{B_c} C_p & \boldsymbol{A_c} & 0 \\ 0 & \boldsymbol{C_c} & -I \end{bmatrix}}_{A} \begin{bmatrix} x_p \\ x_c \\ u \end{bmatrix} \iff E\dot{x} = Ax$$

Beautifully arranged for the use of variable replacement technique!

Derive LMI condition for reset control

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Beautifully arranged for the use of variable replacement technique!

Stability Analysis

$$V = x^T E \bar{P}_i x = x^T \bar{P}_i^{\,T} E x,\ x \in \mathcal{F}_i \qquad \bar{P}_i = \begin{bmatrix} P_i & 0 \\ X & Y \end{bmatrix}$$

Derive LMI condition for reset control

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$$y = C_p x_p$$

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Beautifully arranged for the use of variable replacement technique!

Stability Analysis

$$V = x^T E \bar{P}_i x = x^T \bar{P}_i^T E x, \; x \in \mathcal{F}_i \qquad \bar{P}_i = \begin{bmatrix} P_i & 0 \\ X & Y \end{bmatrix}$$

$$\dot{V} = \dot{x}^T E P_i x + x^T P_i^T E \dot{x} < 0, \; x \in \mathcal{F}_i$$

$$\Leftrightarrow \dot{V} = \begin{bmatrix} E\dot{x} \\ x \end{bmatrix}^T \begin{bmatrix} 0 & P_i \\ P_i^T & \tau_{F_i} M_i \end{bmatrix} \begin{bmatrix} E\dot{x} \\ x \end{bmatrix} < 0$$

By using S-procedure

$$\Leftrightarrow \dot{V} = \begin{bmatrix} E\dot{x} \\ x \end{bmatrix}^T \begin{bmatrix} 0 & P_i \\ P_i^T & \tau_{F_i} M_i \end{bmatrix} \begin{bmatrix} E\dot{x} \\ x \end{bmatrix} + \begin{bmatrix} E\dot{x} \\ x \end{bmatrix}^T G_i \underbrace{[-I \quad A] \begin{bmatrix} E\dot{x} \\ x \end{bmatrix}}_{=0} + \left(\begin{bmatrix} E\dot{x} \\ x \end{bmatrix}^T G_i \underbrace{[-I \quad A] \begin{bmatrix} E\dot{x} \\ x \end{bmatrix}}_{=0} \right)^T < 0$$

Derive LMI condition for reset control

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$$\underbrace{\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{E} \begin{bmatrix} \dot{x}_p \\ \dot{x}_c \\ \dot{u} \end{bmatrix} = \underbrace{\begin{bmatrix} A_p & 0 & B_p \\ \boldsymbol{B_c} C_p & \boldsymbol{A_c} & 0 \\ 0 & \boldsymbol{C_c} & -I \end{bmatrix}}_{A} \begin{bmatrix} x_p \\ x_c \\ u \end{bmatrix} \quad \Leftrightarrow \quad E\dot{x} = Ax$$

Beautifully arranged for the use of variable replacement technique!

Stability Analysis

$$V = x^T E \bar{P}_i x = x^T \bar{P}_i^T E x, x \in \mathcal{F}_i \qquad \bar{P}_i = \begin{bmatrix} P_i & 0 \\ X & Y \end{bmatrix}$$

$$\dot{V} = \dot{x}^T E P_i x + x^T P_i^T E \dot{x} < 0, x \in \mathcal{F}_i$$

$$\Leftrightarrow \dot{V} = \begin{bmatrix} E\dot{x} \\ x \end{bmatrix}^T \begin{bmatrix} 0 & P_i \\ P_i^T & \tau_{F_i} M_i \end{bmatrix} \begin{bmatrix} E\dot{x} \\ x \end{bmatrix} < 0$$

By using S-procedure

$$\Leftrightarrow \dot{V} = \begin{bmatrix} E\dot{x} \\ x \end{bmatrix}^T \begin{bmatrix} 0 & P_i \\ P_i^T & \tau_{F_i} M_i \end{bmatrix} \begin{bmatrix} E\dot{x} \\ x \end{bmatrix} + \begin{bmatrix} E\dot{x} \\ x \end{bmatrix}^T G_i \underbrace{[-I \quad A] \begin{bmatrix} E\dot{x} \\ x \end{bmatrix}}_{=0} + \left(\begin{bmatrix} E\dot{x} \\ x \end{bmatrix}^T G_i \underbrace{[-I \quad A] \begin{bmatrix} E\dot{x} \\ x \end{bmatrix}}_{=0} \right)^T < 0$$

$$\Leftrightarrow \quad \begin{bmatrix} 0 & P_i \\ P_i^T & \tau_{F_i} M_i \end{bmatrix} + He\{G_i [-I \quad A]\} < 0 \qquad \begin{matrix} A_r^T P_N A_r - P_0 + \tau_R M_0 \le 0, \\ \Theta_{i\perp}^T (P_i - P_{i-1}) \Theta_{i\perp} = 0, \end{matrix}$$

Theorem 1

◀ $V(x^+) - V(x) \le 0, \forall x \in \mathcal{J}$
$x^T P_i x = x^T P_j x, \quad \forall x \in X_i \cap X_j$

Derive LMI condition for reset control

- Design condition of reset control based on the stability analysis

$$\begin{bmatrix} 0 & P_i \\ P_i^T & \tau_{F_i} M_i \end{bmatrix} + He\{[-G_i \quad \boldsymbol{G_i A}]\} < 0$$

$$\underbrace{\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{E} \begin{bmatrix} \dot{x}_p \\ \dot{x}_c \\ \dot{u} \end{bmatrix} = \underbrace{\begin{bmatrix} A_p & 0 & B_p \\ \boldsymbol{B_c} C_p & \boldsymbol{A_c} & 0 \\ 0 & \boldsymbol{C_c} & -I \end{bmatrix}}_{A} \begin{bmatrix} x_p \\ x_c \\ u \end{bmatrix}$$

$$E\dot{x} = Ax$$

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$$E\dot{x} = Ax$$

Choose $G_i = \left[\begin{array}{c:c:c} G_{1i} & \begin{matrix} R_1 \\ R_1 \\ R_1 \end{matrix} & \begin{matrix} R_2 \\ R_2 \\ R_2 \end{matrix} \end{array}\right]$

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- Design condition of reset control based on the stability analysis

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$$G_i A = \Omega_i = \begin{bmatrix} G_{1i} & \begin{matrix} R_1 \\ R_1 \\ R_1 \end{matrix} & \begin{matrix} R_2 \\ R_2 \\ R_2 \end{matrix} \end{bmatrix} \begin{bmatrix} A_p & 0 & B_p \\ \boldsymbol{B_c} C_p & \boldsymbol{A_c} & 0 \\ 0 & \boldsymbol{C_c} & -I \end{bmatrix}$$

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$$E\dot{x} = Ax$$

Choose $G_i = \left[\begin{array}{c:c:c} G_{1i} & \begin{matrix} R_1 \\ R_1 \\ R_1 \end{matrix} & \begin{matrix} R_2 \\ R_2 \\ R_2 \end{matrix} \end{array}\right]$

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Nonlinear terms

$R_2 A_c, R_1 B_c, R_2 C_c$

Derive LMI condition for reset control

- Design condition of reset control based on the stability analysis

$$\begin{bmatrix} 0 & P_i \\ P_i^T & \tau_{F_i} M_i \end{bmatrix} + He\{[-G_i \quad \boldsymbol{G_i A}]\} < 0$$

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Choose $G_i = \left[G_{1i} \;\middle|\; \begin{matrix} R_1 \\ R_1 \\ R_1 \end{matrix} \;\middle|\; \begin{matrix} R_2 \\ R_2 \\ R_2 \end{matrix} \right]$

$$G_i A = \Omega_i = \left[G_{1i} \;\middle|\; \begin{matrix} R_1 \\ R_1 \\ R_1 \end{matrix} \;\middle|\; \begin{matrix} R_2 \\ R_2 \\ R_2 \end{matrix} \right] \begin{bmatrix} A_p & 0 & B_p \\ \boldsymbol{B_c} C_p & \boldsymbol{A_c} & 0 \\ 0 & \boldsymbol{C_c} & -I \end{bmatrix}$$

Nonlinear terms

$$R_2 A_c, R_1 B_c, R_2 C_c$$

Replacing

$$\hat{A}_c = R_2 A_c$$
$$\hat{B}_c = R_1 B_c$$
$$\hat{C}_c = R_2 C_c$$

Derive LMI condition for reset control

- Design condition of reset control based on the stability analysis

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$$E\dot{x} = Ax$$

Choose $G_i = \left[\begin{array}{c|c|c} G_{1i} & R_1 & R_2 \\ & R_1 & R_2 \\ & R_1 & R_2 \end{array}\right]$

$$G_i A = \Omega_i = \left[\begin{array}{c|c|c} G_{1i} & R_1 & R_2 \\ & R_1 & R_2 \\ & R_1 & R_2 \end{array}\right] \begin{bmatrix} A_p & 0 & B_p \\ \boldsymbol{B_c} C_p & \boldsymbol{A_c} & 0 \\ 0 & \boldsymbol{C_c} & -I \end{bmatrix}$$

Nonlinear terms

$$R_2 A_c, R_1 B_c, R_2 C_c$$

Replacing

$$\hat{A}_c = R_2 A_c$$
$$\hat{B}_c = R_1 B_c$$
$$\hat{C}_c = R_2 C_c$$

Theorem 2

$$\begin{bmatrix} 0 & P_i \\ P_i^T & \tau_{F_i} M_i \end{bmatrix} + He\{[-G_i \quad \Omega_i]\} < 0$$

where $He\{A\} := A + A^T$

$$A_r^T P_N A_r - P_0 + \tau_R M_0 \le 0,$$
$$\Theta_{i\perp}^T (P_i - P_{i-1}) \Theta_{i\perp} = 0,$$

It is now LMIs

Derive LMI condition for reset control

- Design condition of reset control based on the stability analysis

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$$E\dot{x} = Ax$$

Choose $G_i = \left[\begin{array}{c:c:c} G_{1i} & \begin{matrix} R_1 \\ R_1 \\ R_1 \end{matrix} & \begin{matrix} R_2 \\ R_2 \\ R_2 \end{matrix} \end{array}\right]$

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Nonlinear terms

$$R_2 A_c, R_1 B_c, R_2 C_c$$

Replacing

$$\hat{A}_c = R_2 A_c$$
$$\hat{B}_c = R_1 B_c$$
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It is now LMIs

After solving LMIs,

$$R_2^{-1} \hat{A}_c = A_c$$
$$R_1^{-1} \hat{B}_c = B_c$$
$$R_2^{-1} \hat{C}_c = C_c$$

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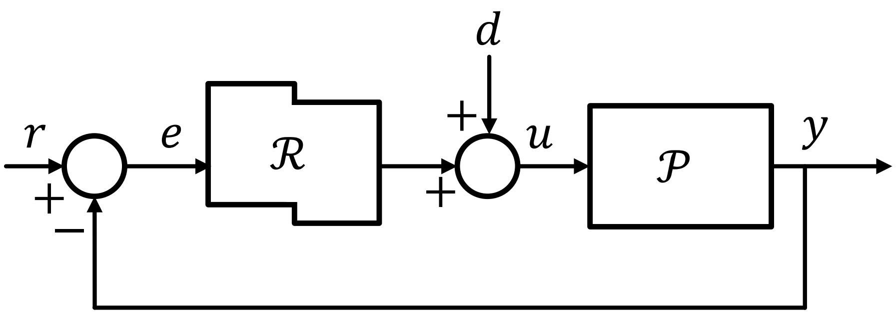
Numerical example

4^{th} order linear plant

$$P(s) = \frac{8s^2 + 18s + 32}{s^4 + 6s^3 + 14s^2 + 24s}$$



By solving the optimization problem

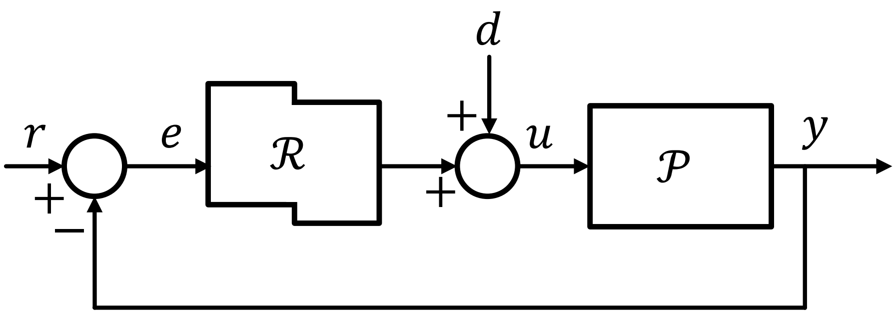
$$\min \gamma$$
subject to
LMIs derived for H_∞ control

where, $\sup \frac{\|y\|_2}{\|d\|_2} \leq \gamma$

Numerical example

4th order linear plant

$$P(s) = \frac{8s^2 + 18s + 32}{s^4 + 6s^3 + 14s^2 + 24s}$$



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LMIs derived for H_∞ control

where, $\sup \frac{\|y\|_2}{\|d\|_2} \leq \gamma$

$$A_c = \begin{bmatrix} -2.8978 & -0.8035 & -3.2265 & -52.7260 \\ 2.1708 & 0.4710 & 3.4589 & 14.4138 \\ -0.3965 & -0.5404 & -1.5177 & -3.1082 \\ 0.0035 & 0.0044 & 0.0047 & -0.8351 \end{bmatrix}$$

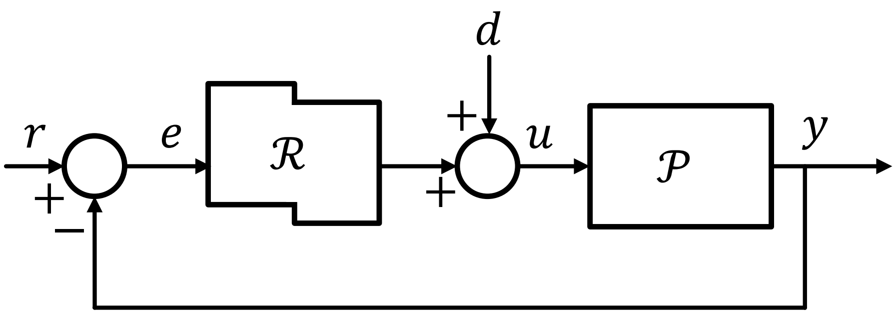
$$B_c = \begin{bmatrix} -0.9634 & 2.9398 & -1.7962 & -0.2300 \end{bmatrix}^T$$

$$C_c = [0 \quad 0 \quad 0 \quad 1]$$

Numerical example

4th order linear plant

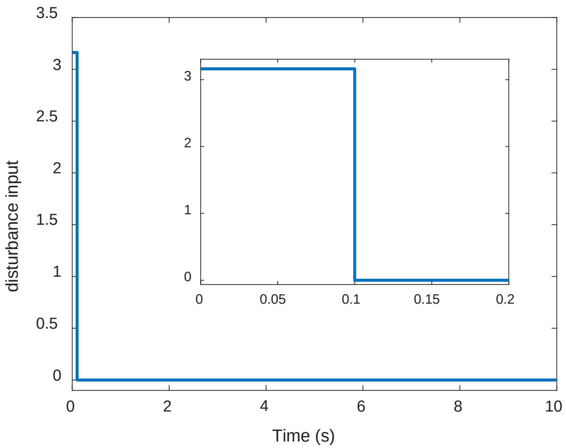
$$P(s) = \frac{8s^2 + 18s + 32}{s^4 + 6s^3 + 14s^2 + 24s}$$



By solving the optimization problem

$$\min \gamma$$
subject to
LMIs derived for H_∞ control

where, $\sup \frac{\|y\|_2}{\|d\|_2} \leq \gamma$



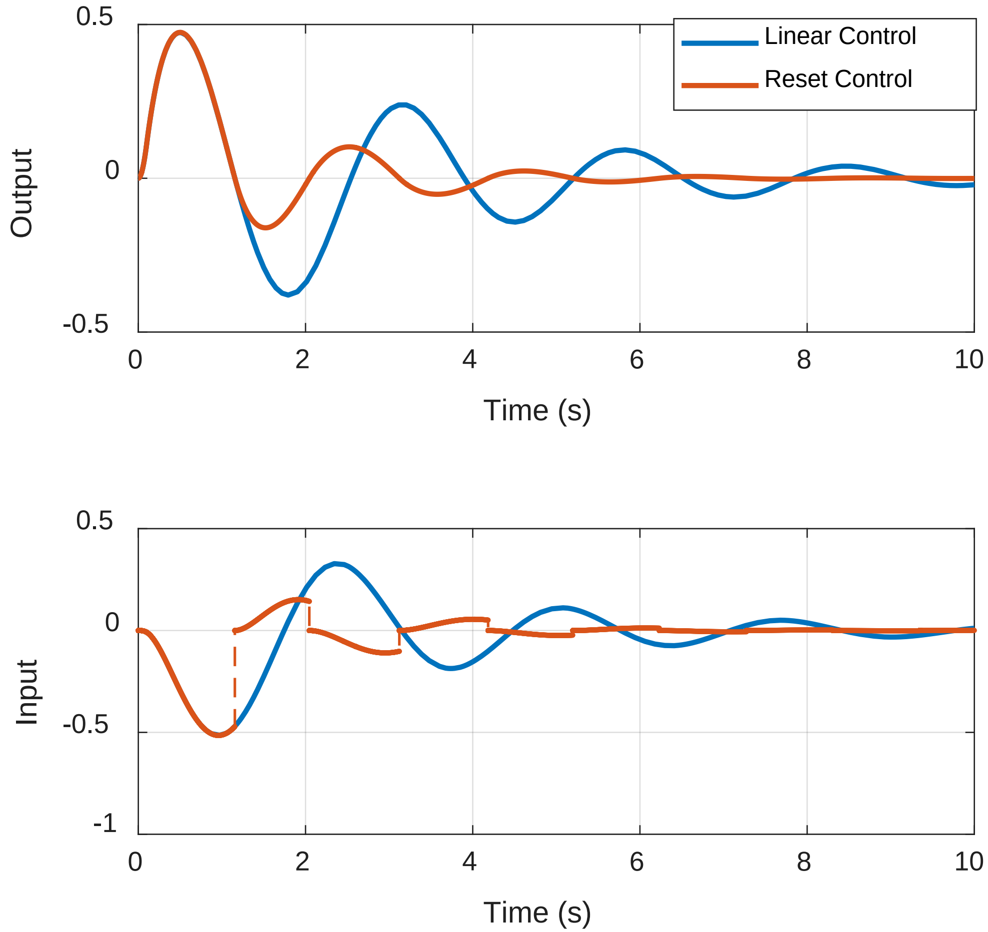
$$A_c = \begin{bmatrix} -2.8978 & -0.8035 & -3.2265 & -52.7260 \\ 2.1708 & 0.4710 & 3.4589 & 14.4138 \\ -0.3965 & -0.5404 & -1.5177 & -3.1082 \\ 0.0035 & 0.0044 & 0.0047 & -0.8351 \end{bmatrix}$$

$$B_c = \begin{bmatrix} -0.9634 & 2.9398 & -1.7962 & -0.2300 \end{bmatrix}^T$$

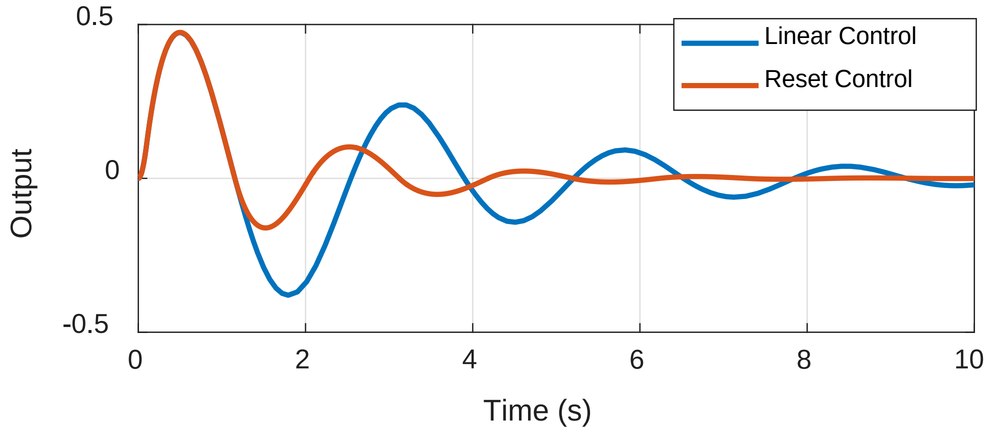
$$C_c = [0 \quad 0 \quad 0 \quad 1]$$

$$d = \begin{cases} \sqrt{10} & 0 \leq t < 0.1 \\ 0 & 0.1 \leq t \end{cases}$$

Numerical example



Numerical example



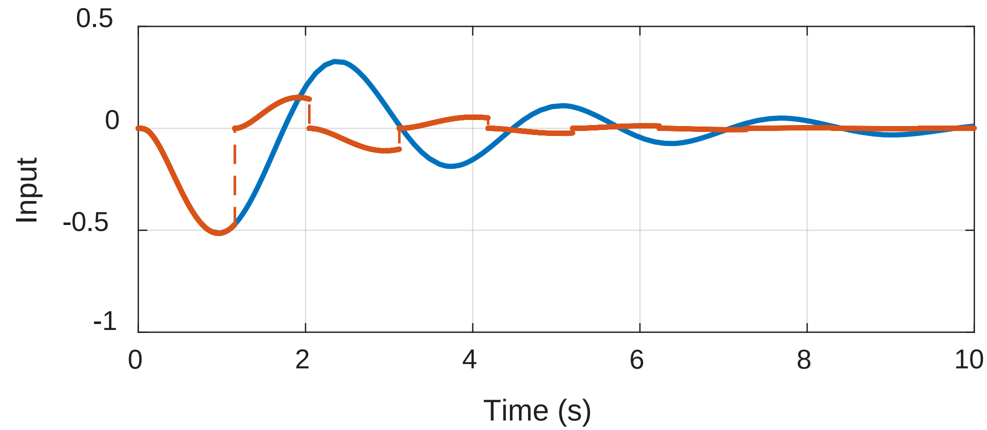


Table. Comparison of $\mathcal{L}_2$ gains

	$\sup \frac{\|y\|_2}{\|d\|_2}$
Linear control	0.5370
Reset Control	**0.3833**

28.61% decreased by reset control

Conclusions

- LMI-based design of reset control is addressed
- Previous stability condition yields BMI design condition
- Formulated closed-loop system as a descriptor hybrid system
- Proposed two theorems
  - The stability analysis of descriptor hybrid system
  - LMI-based design of reset control system
- Numerical example showed the feasibility of the design strategy

Thank you for listening!
Q&A

Appendix

- A linear matrix inequality (LMI) in the variable $x \in \mathbb{R}^n$ has the form

$$F(x) \coloneqq F_0 + x_1 F_1 + x_2 F_2 + \cdots + x_n F_n \succcurlyeq 0,$$

where $F_0 \in \mathbb{R}^{m \times m}, \ldots, F_n \in \mathbb{R}^{m \times m}$ are symmetric matrices.

Example of LMIs (Lyapunov stability)

$$A^T \boldsymbol{P} + \boldsymbol{P} A < 0, P > 0$$

where, P is matrix variables. (A is stable if and only if there exist such P.)

For $P = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix}, A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},$

$$p_1 \begin{bmatrix} 2a_{11} & a_{12} \\ a_{12} & 0 \end{bmatrix} + p_2 \begin{bmatrix} 2a_{21} & a_{11} + a_{22} \\ a_{11} + a_{22} & 2a_{12} \end{bmatrix} + p_3 \begin{bmatrix} 0 & a_{21} \\ a_{21} & 2a_{22} \end{bmatrix} < 0$$

$$p_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + p_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + p_3 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} > 0$$

Appendix

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